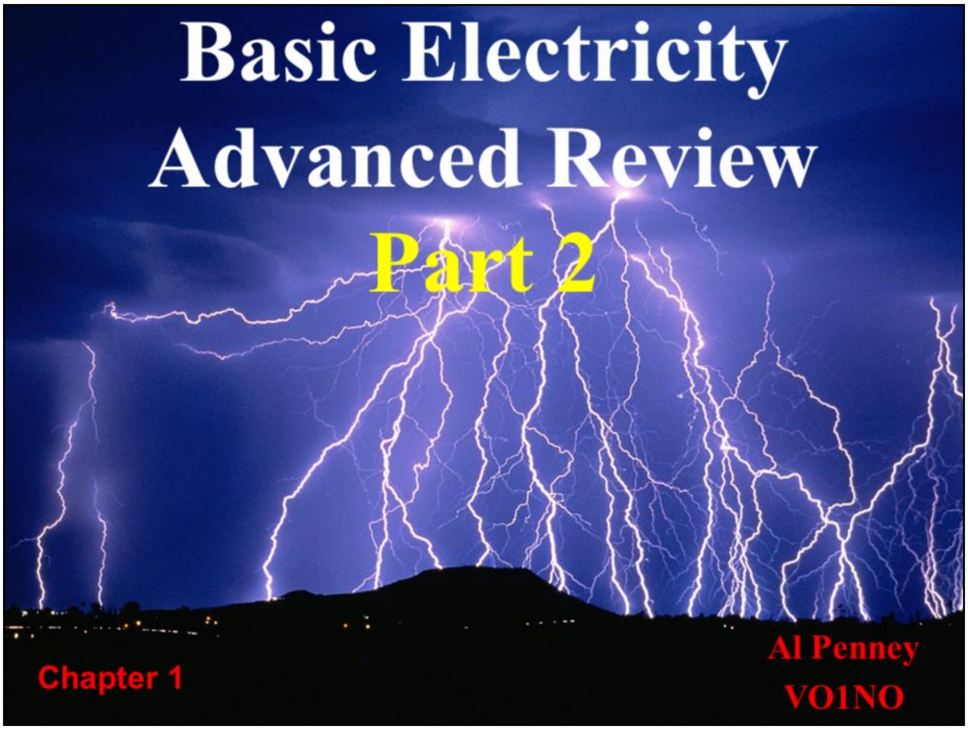
Basic Electricity
Advanced Review
Part 2
Chapter 1
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Inductance

- **Inductance** is the property of an electrical circuit that **opposes a change in current.**
- In a **DC circuit** inductance has **an effect only** when the **DC starts**, or when **attempts are made to stop it.**
- In an **AC circuit** though, the **voltage** is **constantly changing,** and **inductance** constantly **works to retard** the **change in current.**

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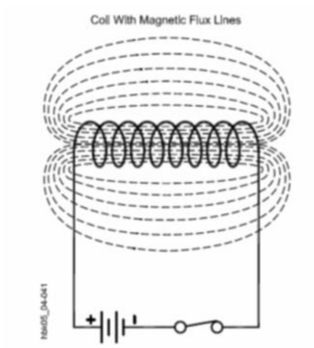
Current Through a Wire

- A **current** through a **wire** will generate a **magnetic field** around that wire, as can be demonstrated by bringing a **compass** near that wire.



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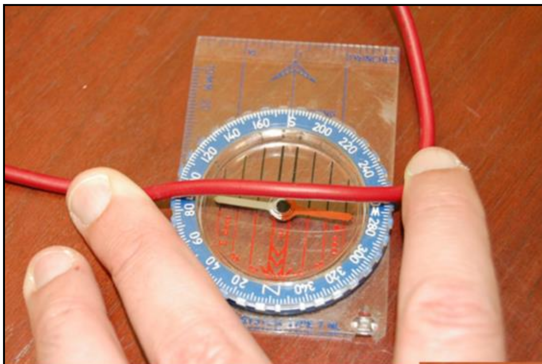
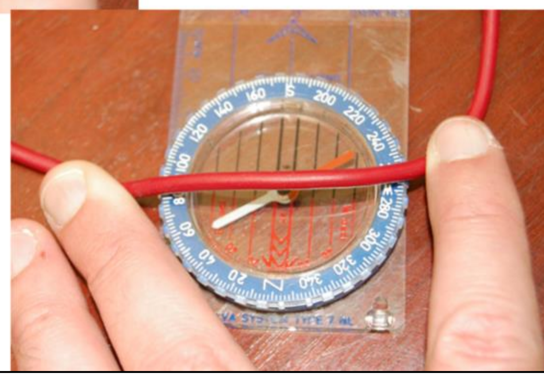


No Current

Current



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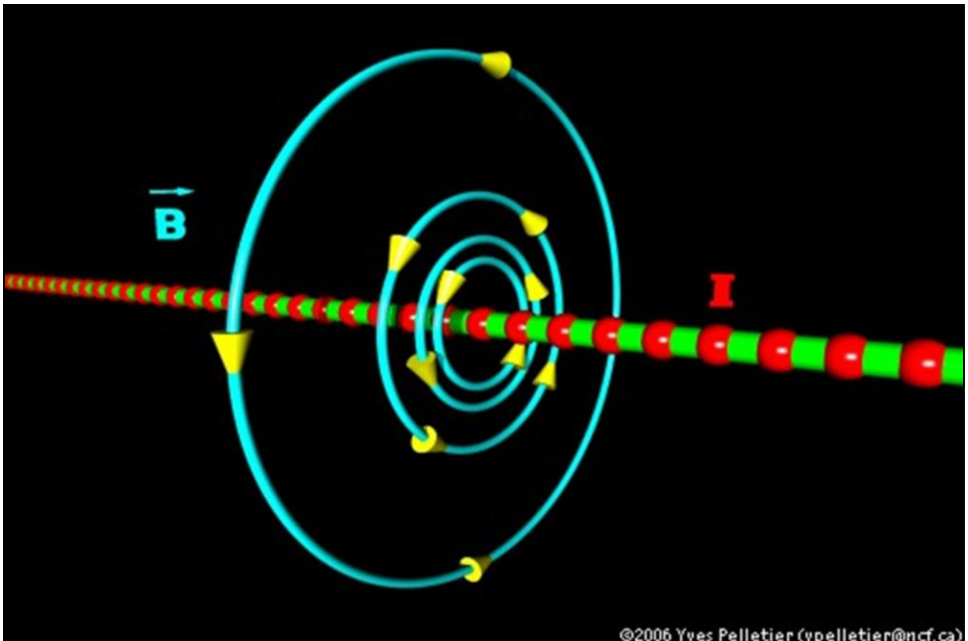

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Magnetic Field Effects on a Wire

- Conversely, when magnetic **lines of flux cut** through a **wire,** a **current** will be **induced** to flow in that wire.
- This is the **basis for generators.**

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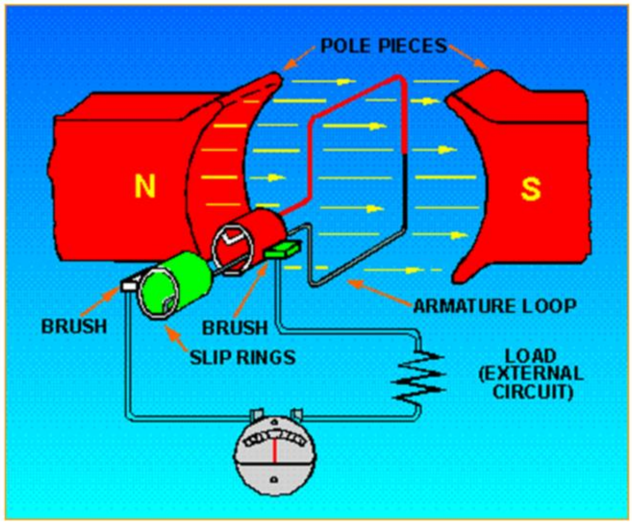
Elementary Generator



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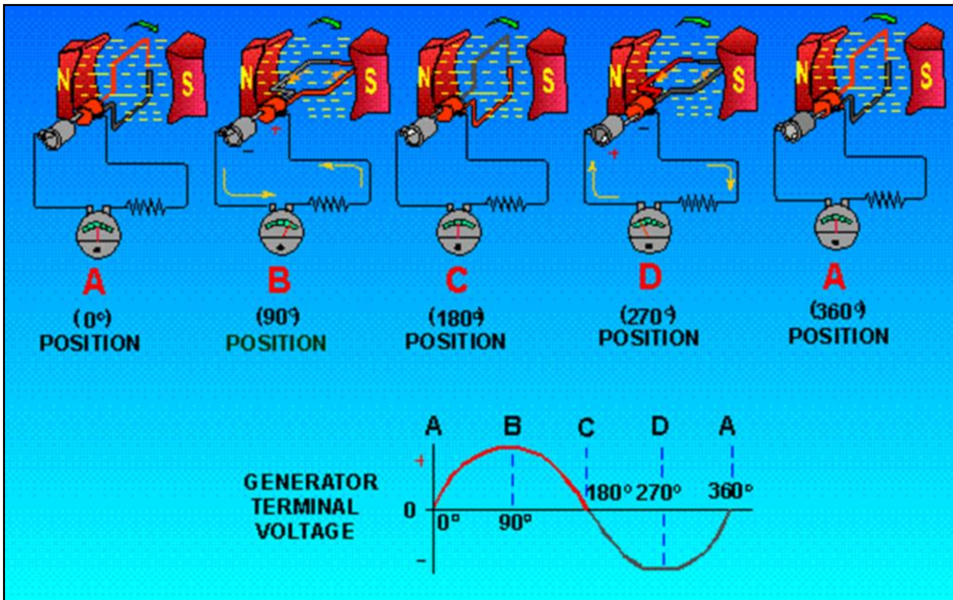

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Back EMF

- When a current **starts to flow** through a wire, it takes a **finite time** for the **magnetic field** to build up to its **final size.**
- As the magnetic field builds up, its **own lines of flux cut through the conductor.**
- This **induces** a **voltage** and **resulting current** in that wire.
- Because of Conservation of Energy reasons, that **induced current opposes the applied current.**
- This **opposing voltage** is called the **Counter** or **Back EMF (Electro Motive Force** – voltage).

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Inductor in a DC Circuit

- **Counter EMF** can **only** be generated as the **magnetic field** around a conductor is **changing.**
- **After the initial current surge** in a DC circuit, the **current**, and therefore the **magnetic field, stabilize and remain steady.**
- The Counter EMF therefore **disappears.**
- Usually, **inductance** can be **ignored** in **most DC circuits,** however…

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Back EMF Backlash!

- In some devices such as **electric motors** and **relays**, the **Counter EMF** can **cause problems.**
- When the device is **turned off,** the **magnetic field collapses,** inducing a **strong Back EMF.**
- This can be strong enough that it can cause an **arc in the switch** that controls the device.
- Sometimes it can even **weld the switch shut**, restarting the device and making it very difficult to stop.

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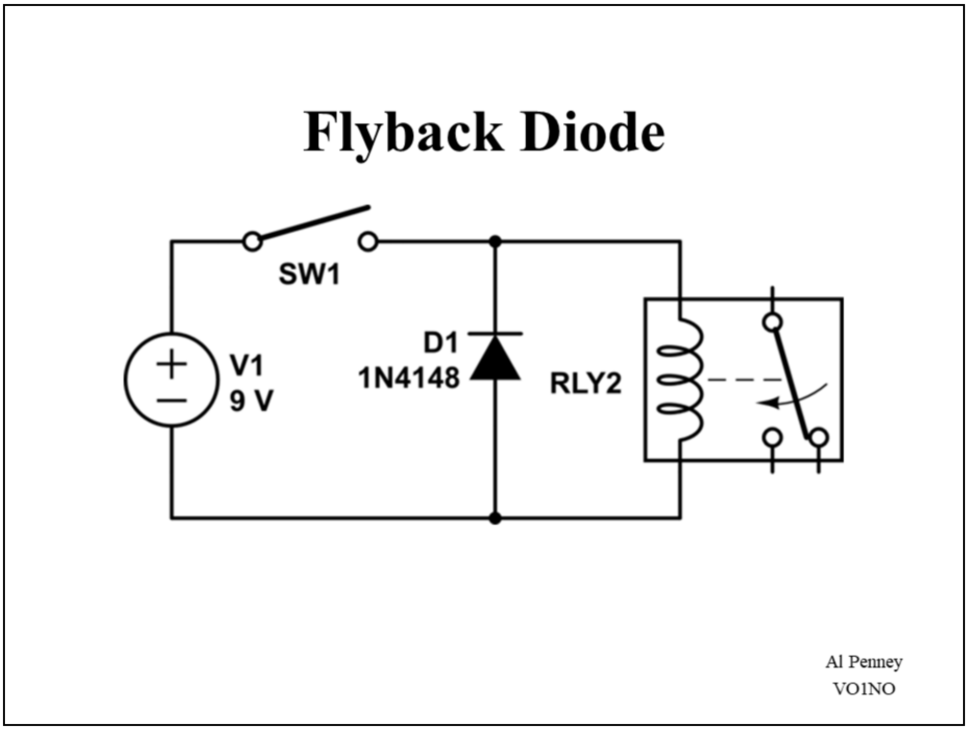

A flyback diode, is a diode that is placed with reverse polarity from the power supply and in parallel to the relay's inductance coil. It is used to prevent the huge voltage spikes that happen when the power supply is disconnected. They are sometimes called "snubber diodes" and are a type of snubber circuit.

When the power supply is connected to the relay, the inductance coil's voltage builds up to match that of the power source. The speed at which current can change in an inductor is limited by its time constant. In this case, the time it takes to minimize current flow through the coil is longer than the time it takes for the power supply to be removed. Upon disconnection, the inductance coil reverses its polarity in an attempt to keep current flowing according to its dissipation curve (i.e., % of maximum current flow with respect to time). This causes a huge voltage potential to build up on the open junctions of the component that controls the relay.

This voltage built up is called flyback voltage. It can result in an electrical arc and damage the components controlling the relay. It can also introduce electrical noise that can couple into adjacent signals or power connections and cause microcontrollers to crash or reset. If you have an electronics control panel that resets each time a relay is de-energized, it’s highly possible you have an issue with flyback voltage.

To mitigate this issue, a diode is connected with reverse polarity to the power supply. No current passes through the diode when the relay is energized. When the power supply is removed, the voltage polarity on the coil is inverted, and the diode becomes forward biased. The diode allows current to pass with minimal resistance and prevents flyback voltage from building up. Hence why it is called a flyback diode.

Inductor in an AC Circuit

- In an **AC circuit,** the **voltage,** and therefore the **current,** is **constantly changing.**
- Because of this, the **magnetic field** around the conductor carrying the current is **constantly changing** as well.
- As the **magnetic field** alternately **expands outwards** and **collapses inwards,** the **magnetic lines of flux** are constantly **cutting** through the conductor.
- This creates a Counter EMF that constantly **acts to oppose any change in current.**

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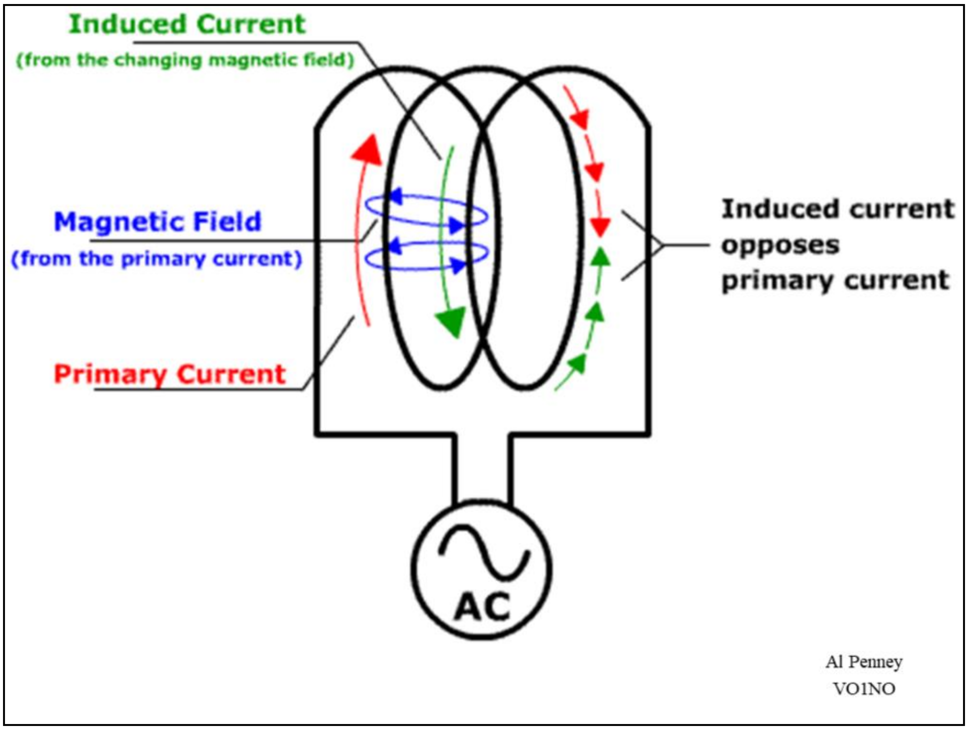
Induced Current
(from the changing magnetic field)
Magnetic Field
(from the primary current)
Primary Current
Induced current
opposes
primary current
AC
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The henry

- The **unit of measurement** for inductance is the **henry,** abbreviated **"L".**
- An inductor is said to have an inductance of 1 henry if a **current** passing through it at a rate of **1 ampere per second** causes a **Counter EMF** of **1 volt** to be generated.
- This is **too large a unit** for most applications however, so **millihenrys (mh)** or **microhenrys (µh)** are more commonly encountered in electronic equipment.

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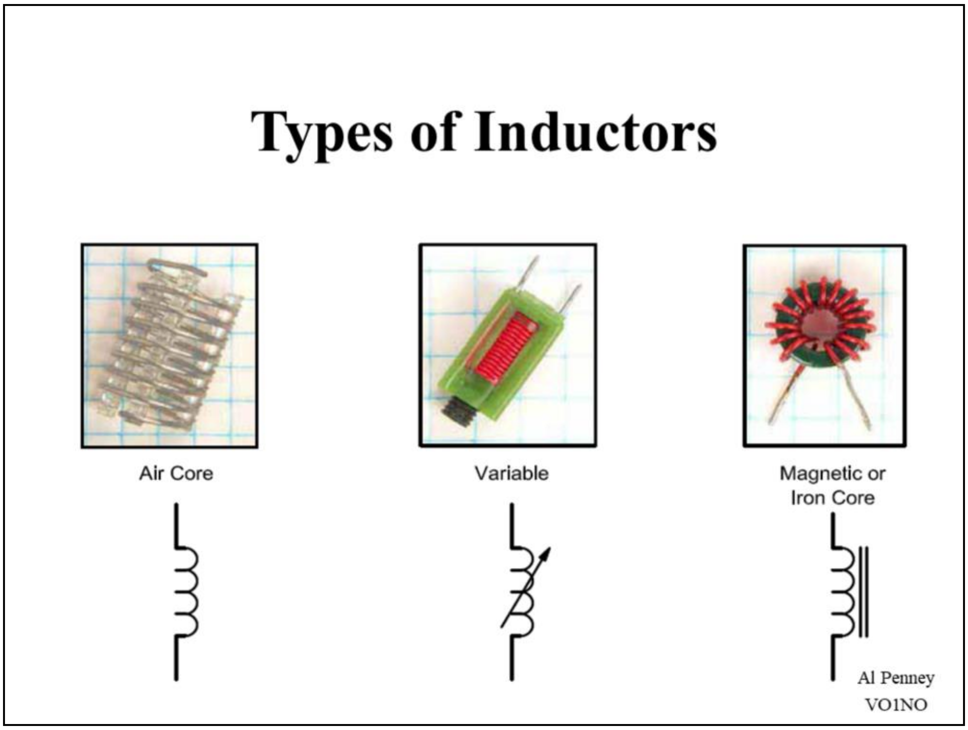
Types of Inductors
Air Core
Variable
Magnetic or
Iron Core
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Roller Inductor

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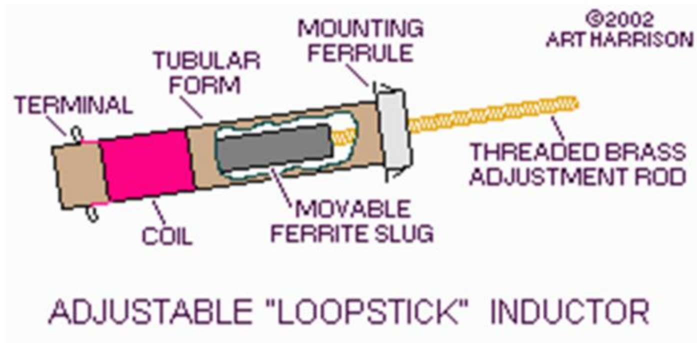
Loopstick Inductor



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Factors Affecting Inductance

- **Number of Turns:** The inductance of a coil is proportional to the **square of the number of turns.**
- A coil with **twice the number of turns** as another otherwise identical coil will have **four times the inductance.** A coil with **3 times as many turns** will have **9 times the inductance.**

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Factors Affecting Inductance

- **Coil Diameter:** The **larger the diameter** of the coil, the **greater the inductance.**
- A **coil** with **twice the diameter** of an otherwise identical coil will have **twice the inductance.**

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Factors Affecting Inductance

- **Changing the core:** Certain materials will **concentrate the lines of magnetic flux** better than others, and will therefore **increase the inductance** if used as a **core** for the coil.
- For example, a coil wound on an **iron core** will have much **more inductance** than one with an **air core.**

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Core Materials

Properties of Some High-Permeability Materials

Material	*Approximate Percent Composition*					*Maximum Permeability*
	Fe	Ni	Co	Mo	Other	
Iron	99.91	—	—	—	—	5000
Purified Iron	99.95	—	—	—	—	180000
4% silicon-iron	96	—	—	—	4 Si	7000
45 Permalloy	54.7	45	—	—	0.3 Mn	25000
Hipernik	50	50	—	—	—	70000
78 Permalloy	21.2	78.5	—	—	0.3 Mn	100000
4-79 Permalloy	16.7	79	—	—	0.3 Mn	100000
Supermalloy	15.7	79	—	5	0.3 Mn	800000
Permendur	49.7	—	50	—	0.3 Mn	5000
2V Permendur	49	—	49	—	2 V	4500
Hiperco	64	—	34	—	2 Cr	10000
2-81 Permalloy*	17	81	—	2	—	130
Carbonyl iron*	99.9	—	—	—	—	132
Ferroxcube III**	($MnFe_2O_4$ + $ZnFe_2O_4$)				1500	

Note: all materials in sheet form except * (insulated powder) and ** (sintered powder).
(Reference: L. Ridenour, ed., *Modern Physics for the Engineer*, p 119.)

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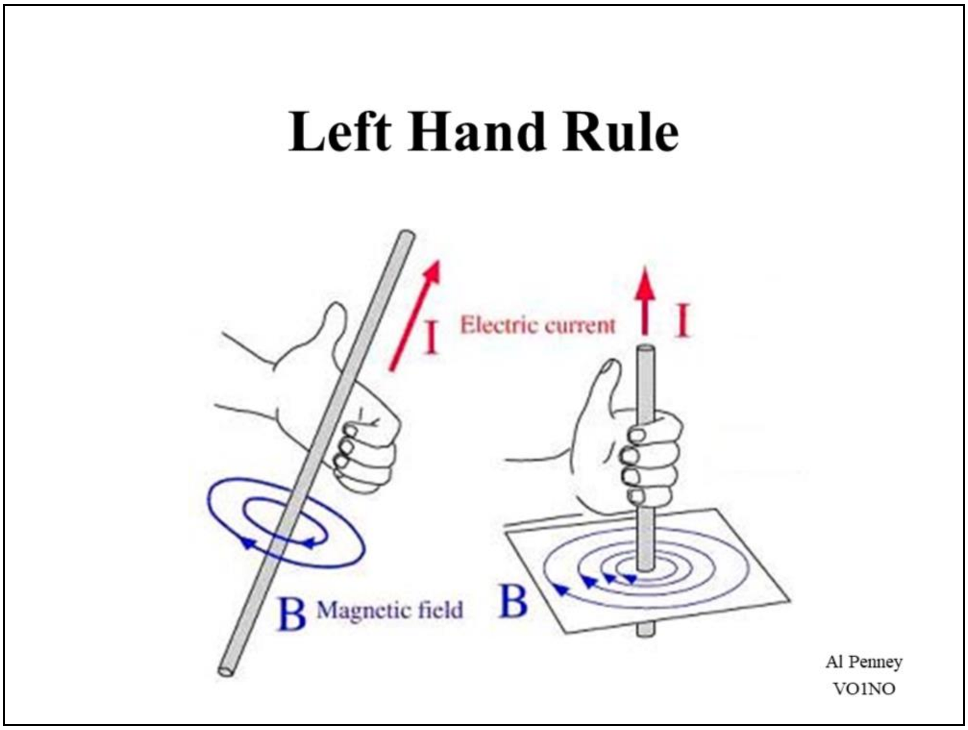
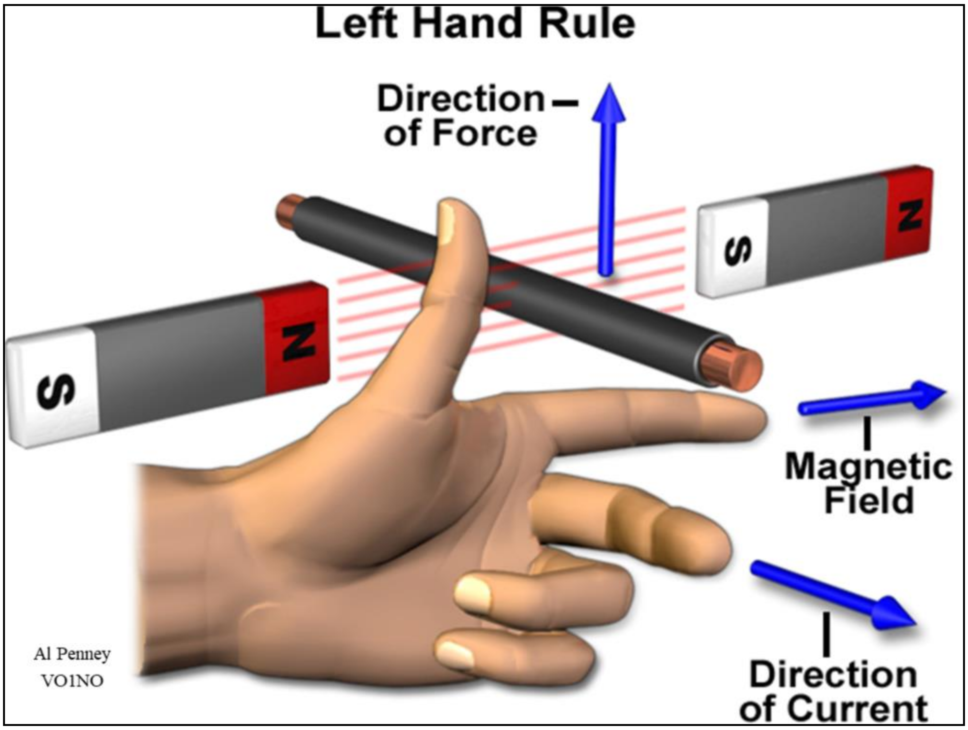


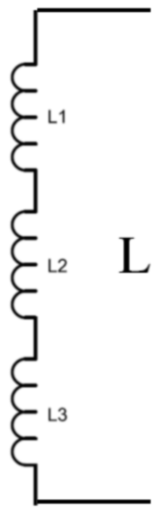
Pretend you are wrapping your fingers around a thin rod (in other words, make a fist) and point your thumb in the direction of the current (I). The magnetic field will circle around your fist like in the following diagram. Additionally, the magnetic field will always point in the direction your fingers are curled. Use the following diagram for reference to both the new vectors and the first left hand rule.



Whenever a current carrying conductor comes under a magnetic field, there will be a force acting on the conductor. The direction of this force can be found using Fleming's Left Hand Rule (also known as 'Flemings left-hand rule for motors').

It is found that whenever a current carrying conductor is placed inside a magnetic field, a force acts on the conductor, in a direction perpendicular to both the directions of the current and the magnetic field.

Inductors in Series



$$L_{total} = L1 + L2 + L3 \ldots + L_n$$

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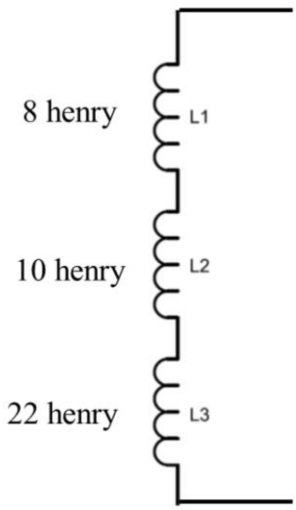
Example - Inductors in Series



$$L_{total} = L1 + L2 + L3$$

$$L_{total} = 8H + 10H + 22H$$

$$L_{total} = 40\ H$$

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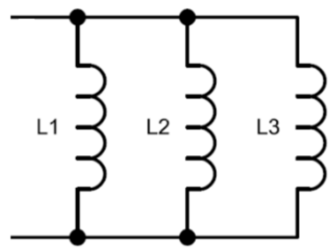
Inductors in Parallel

$$L_{total} = \frac{1}{\frac{1}{L1} + \frac{1}{L2} + \frac{1}{L3} + \ldots + \frac{1}{L_n}}$$



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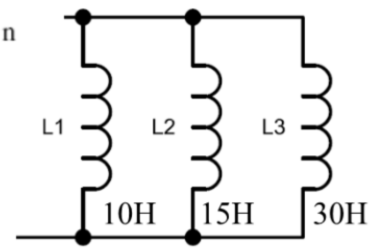
Example - Inductors in Parallel

$$L_{total} = \frac{1}{\frac{1}{L1} + \frac{1}{L2} + \frac{1}{L3} + \ldots + \frac{1}{L_n}}$$



$$L_{total} = \frac{1}{\frac{1}{10} + \frac{1}{15} + \frac{1}{30}}$$

$$L_{total} = \frac{1}{\frac{3}{30} + \frac{2}{30} + \frac{1}{30}} = \frac{1}{6/30} = 5H$$

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Reactance

- **Reactance** is the **opposition** to the **flow of Alternating Current (AC).**
- **Reactance** has **no effect** on the flow of **Direct Current (DC).**

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Inductive Reactance

- **Inductive Reactance** is the **opposition** to the **flow of current** in an **AC circuit** caused by an **inductor.**
- As the **frequency increases,** Inductive Reactance **also increases.**
- The **symbol** for **Inductive Reactance** is $\mathbf{X_L}$.
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy **stored in an inductor's magnetic field** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

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Inductive Reactance

$$X_L = 2 \pi f L$$

- Where:

  f = frequency in Hertz

  L = inductance in henrys

  π = 3.14

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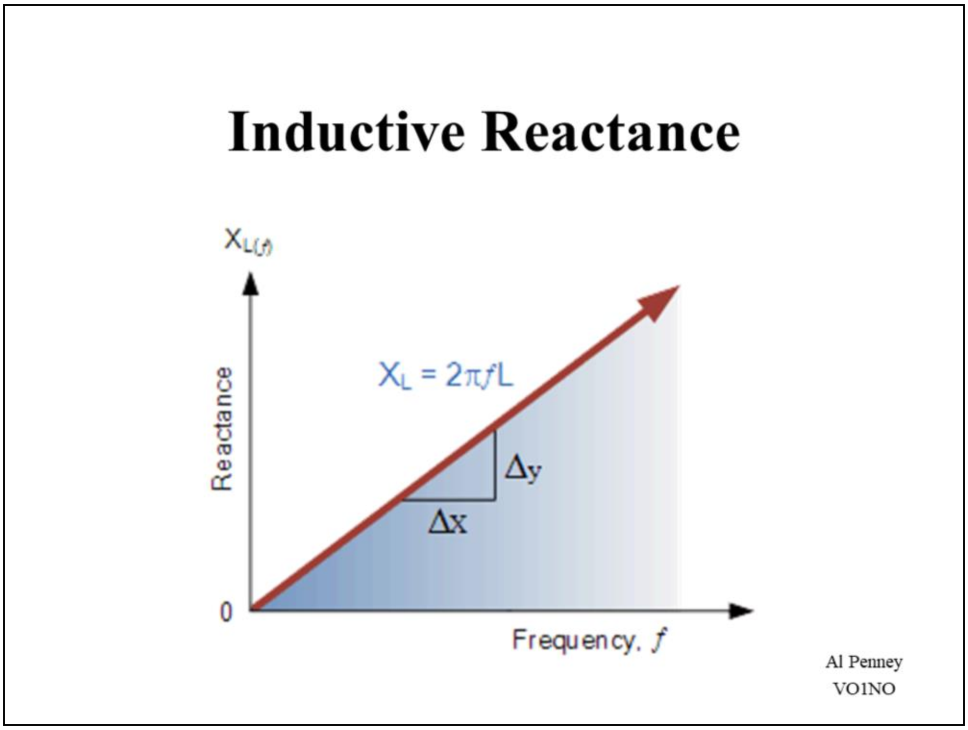

From the above equation for inductive reactance, if either the **Frequency** or the **Inductance** is increased the overall inductive reactance value of the inductor would also increase. As the frequency approaches infinity the inductors reactance would also increase towards infinity with the circuit element acting like an open circuit.

However, as the frequency approaches zero or DC, the inductors reactance would decrease to zero, causing the opposite effect acting like a short circuit. This means then that inductive reactance is "**Proportional**" to frequency and is small at low frequencies and high at higher frequencies and this demonstrated in the following curve:

The graph of inductive reactance against frequency is a straight line linear curve. The inductive reactance value of an inductor increases linearly as the frequency across it increases. Therefore, inductive reactance is positive and is directly proportional to frequency ($X_L \propto f$)

Inductive Reactance Example 1

- What is the reactance of a coil having an inductance of 8.00 henrys at a frequency of 120 Hertz?

$$X_L = 2 \pi f L$$

$$\mathbf{X_L = 2 \times 3.14 \times 120\ Hertz \times 8.00H}$$

$$\mathbf{X_L = 6030\ Ohms}$$

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Inductive Reactance Example 2

- What is the reactance of that same coil having an inductance of 8.00 henrys at a frequency of 2 kHz?

$$X_L = 2 \pi f L$$

Remember that **2 kHz = 2000 Hz**

X_L = 2 x 3.14 x 2000 Hertz x 8.00H

X_L = 100,480 Ohms

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Inductive Reactance Examples

- Note that as the **frequency increased** from 120 Hz to 2000 Hz, the **Inductive Reactance increased** from 6030 ohms to 100,480 ohms.
- **Remember:**
  - **Inductors allow DC to pass, but hinder AC;**
  - Inductors **store energy** as a magnetic field; and
  - As the **frequency increases, inductive reactance increases (and vice versa!).**

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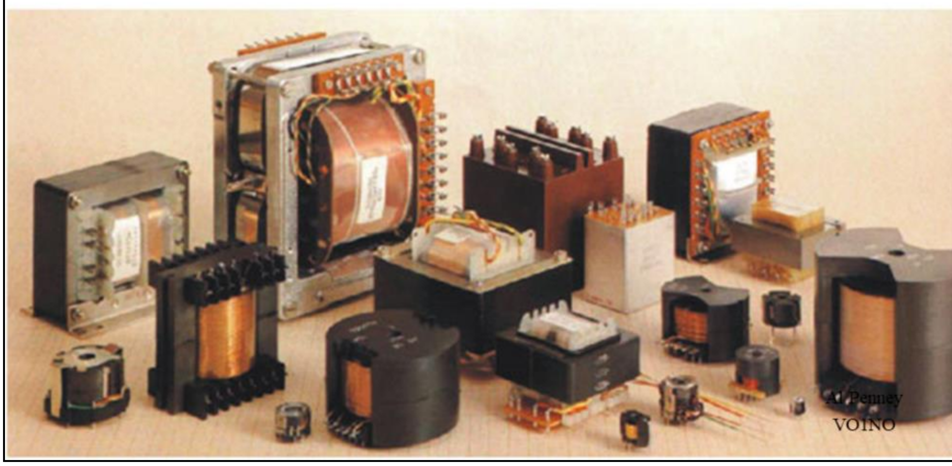
Transformers



Transformers

- Any device that **transfers power** from **one voltage-current level** to **another voltage-current level** is called a **transformer.**
- Transformers work on the principle of **changing current** in **one inductor** inducing a **current** in **another inductor.**

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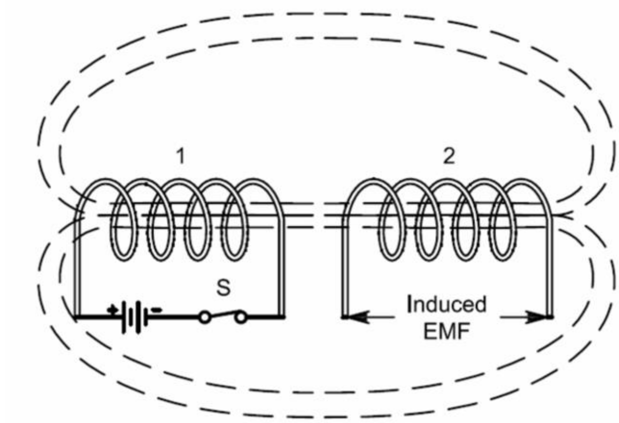
Induced EMF



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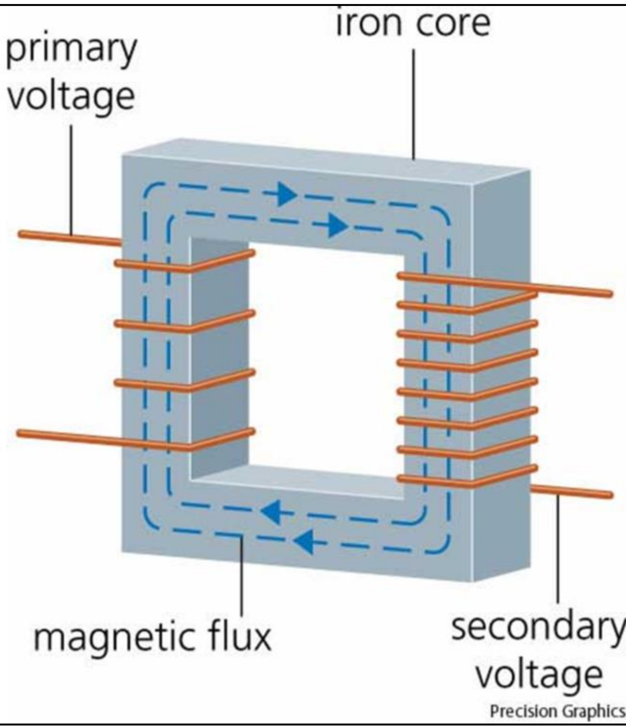

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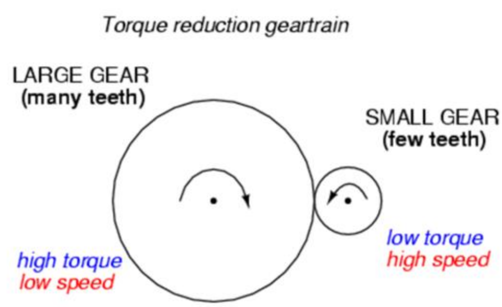
Transformer Mechanical Equivalent



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Transformer Applications

- Transformers have **3 primary applications:**
  - **Isolating** one part of a circuit from another (**magnetic** linkage only, versus **conductive** linkage);
  - **Stepping voltages up or down**; and
  - **Impedance matching.**

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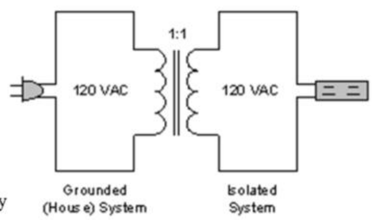
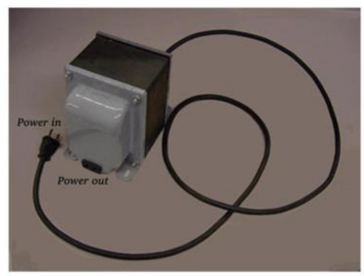
Isolation Transformer

- Many uses for isolation transformers in electronic circuits.
- Also used in power circuits, using transformers that have a 1:1 turns ratio.





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Audio systems often use 1:1 transformers to isolate one part of a circuit from another e.g. interfaces between a radio's audio/mic and a computer's mic/speaker connections when using digital modes.

Changing the Voltage

- A **transformer** can be used to **step the voltage up or down.**
- The **ratio of turns** in the primary and secondary windings determine the **amount of voltage change:**

$$\frac{\text{Primary Voltage}}{\text{Secondary Voltage}} = \frac{\text{\# Turns Primary winding}}{\text{\# Turns Secondary winding}}$$

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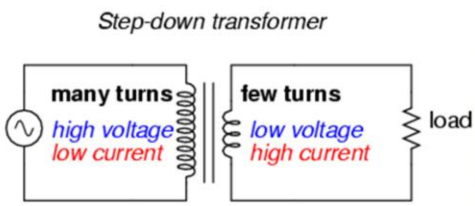
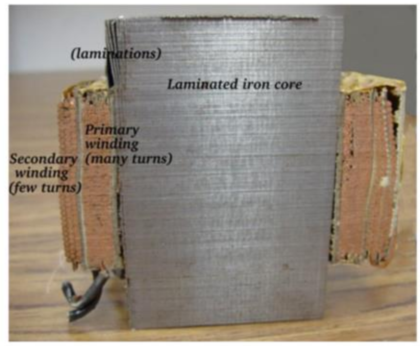
Step Down Transformer





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Impedance Matching

- Transformers are used to **match differing impedances** in RF and AF circuits.
- The **turns ratio** determines the **degree of impedance change.**

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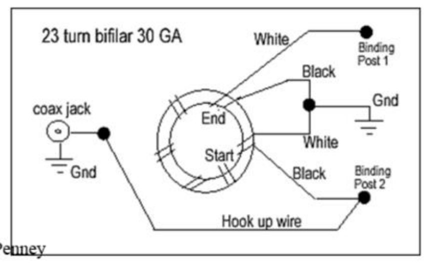
Antenna Impedance Matching

- Transformers are often used to **match impedances** in **antenna systems.**
- The most frequently encountered are **1:1** and **4:1,** but other impedance transformations are available.



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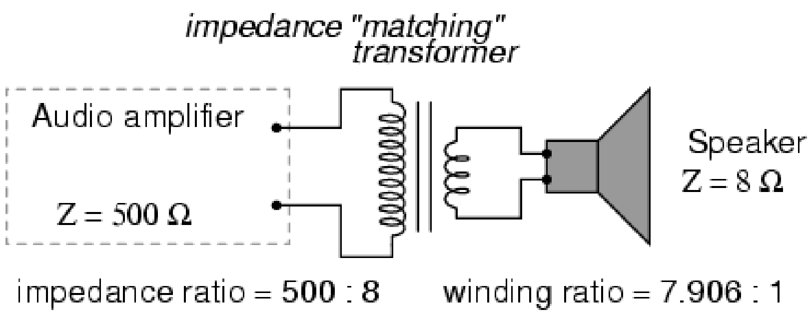
Audio Impedance Matching



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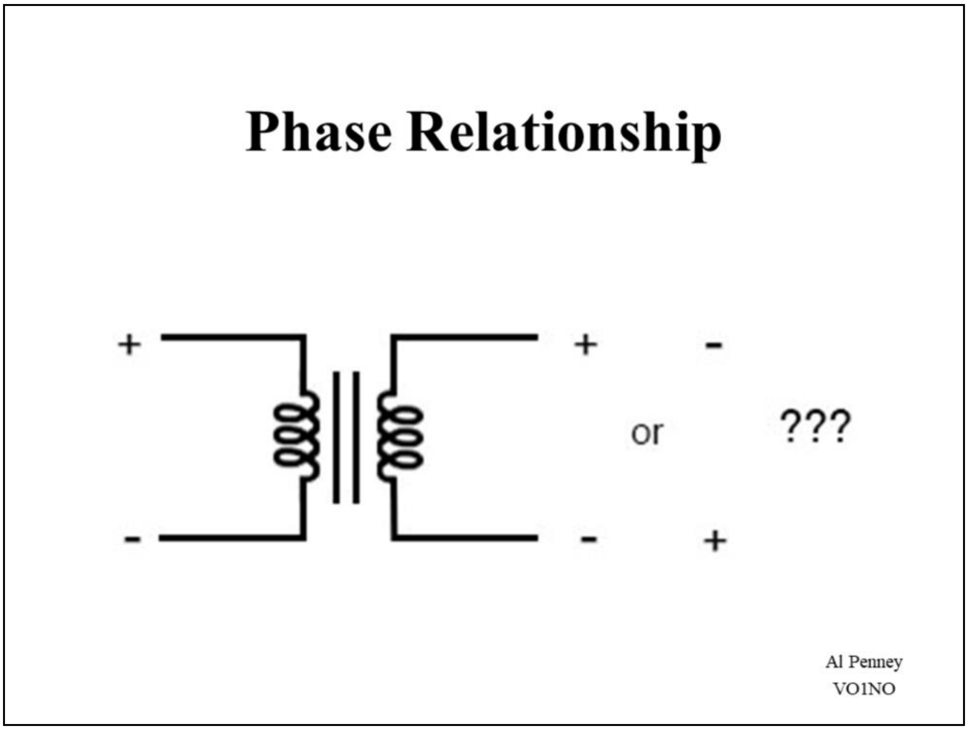
To obtain an impedance transformation ratio of 500:8, we would need a winding ratio equal to the square root of 500:8 (the square root of 62.5:1, or 7.906:1). With such a transformer in place, the speaker will load the amplifier to just the right degree, drawing power at the correct voltage and current levels to satisfy the Maximum Power Transfer Theorem and make for the most efficient power delivery to the load. The use of a transformer in this capacity is called *impedance matching*.

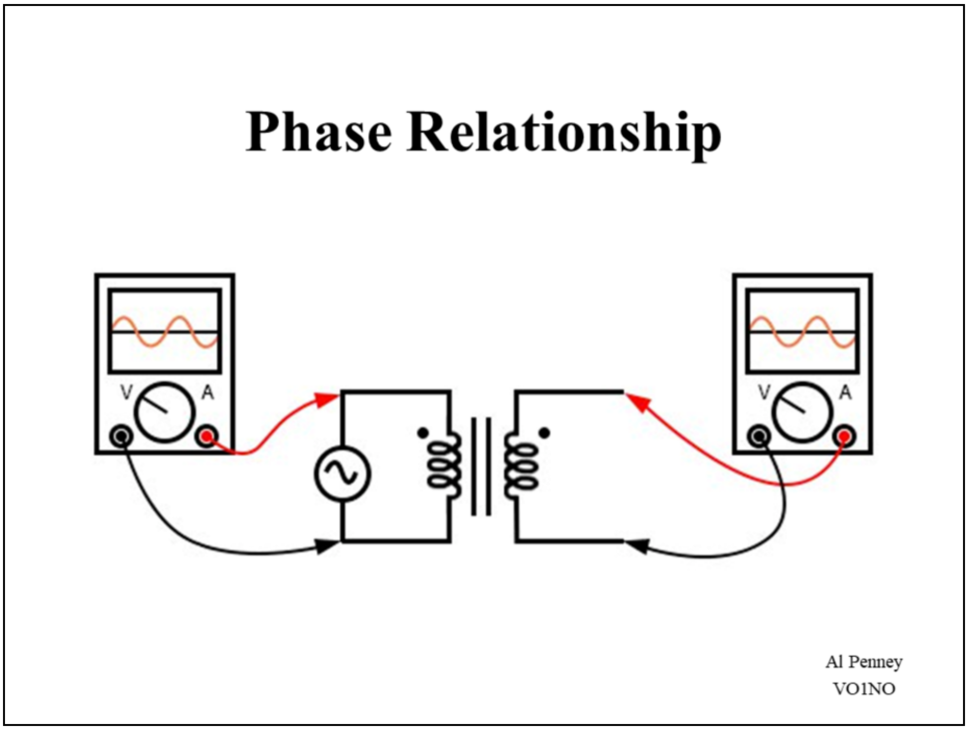
Anyone who has ridden a multi-speed bicycle can intuitively understand the principle of impedance matching. A human's legs will produce maximum power when spinning the bicycle crank at a particular speed (about 60 to 90 revolution per minute). Above or below that rotational speed, human leg muscles are less efficient at generating power. The purpose of the bicycle's "gears" is to impedance-match the rider's legs to the riding conditions so that they always spin the crank at the optimum speed.

If the rider attempts to start moving while the bicycle is shifted into its "top" gear, he or she will find it very difficult to get moving. Is it because the rider is weak? No, it's because the high step-up ratio of the bicycle's chain and sprockets in that top gear presents a mismatch between the conditions (lots of inertia to overcome)

and their legs (needing to spin at 60-90 RPM for maximum power output). On the other hand, selecting a gear that is too low will enable the rider to get moving immediately, but limit the top speed they will be able to attain. Again, is the lack of speed an indication of weakness in the bicyclist's legs? No, it's because the lower speed ratio of the selected gear creates another type of mismatch between the conditions (low load) and the rider's legs (losing power if spinning faster than 90 RPM). It is much the same with electric power sources and loads: there must be an impedance match for maximum system efficiency. In AC circuits, transformers perform the same matching function as the sprockets and chain ("gears") on a bicycle to match otherwise mismatched sources and loads.



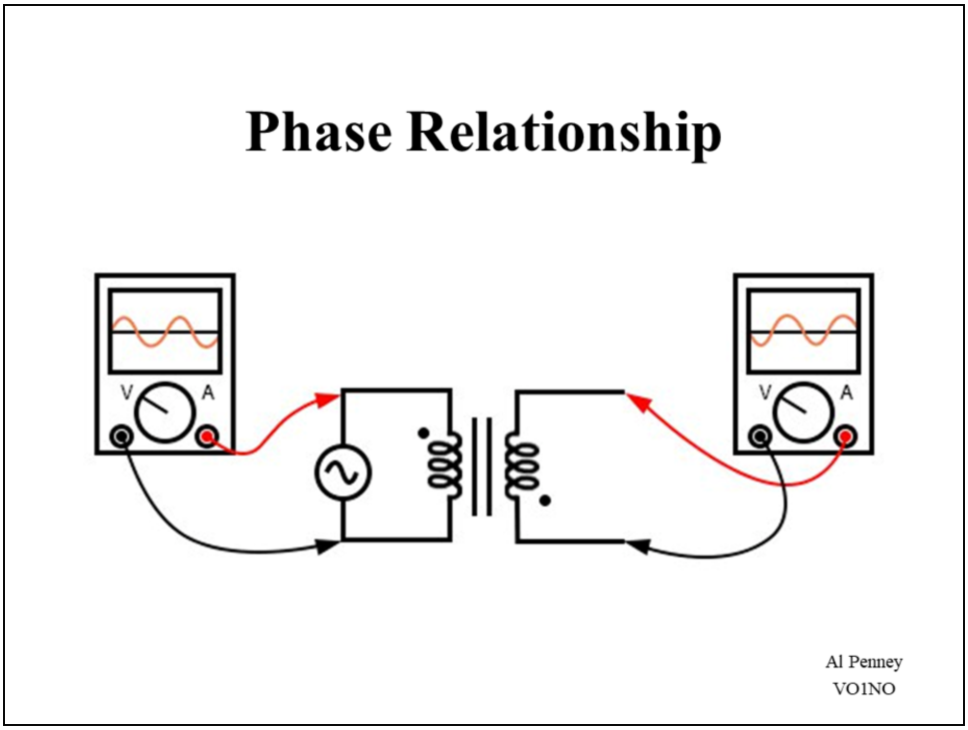
It would appear that both voltage and current for the two transformer windings are in-phase with each other, at least for our resistive load. This is simple enough, but it would be nice to know *which way* we should connect a transformer in order to ensure the proper phase relationships be kept. After all, a transformer is nothing more than a set of magnetically-linked inductors, and inductors don't usually come with polarity markings of any kind. If we were to look at an unmarked transformer, we would have no way of knowing which way to hook it up to a circuit to get in-phase (or 180° out-of-phase) voltage and current:



Since this is a practical concern, transformer manufacturers have come up with a sort of polarity marking standard to denote phase relationships. It is called the *dot convention*, and is nothing more than a dot placed next to each corresponding leg of a transformer winding:

Typically, the transformer will come with some kind of schematic diagram labeling the wire leads for primary and secondary windings. On the diagram will be a pair of dots similar to what is seen above. Sometimes dots will be omitted, but when "H" and "X" labels are used to label transformer winding wires, the subscript numbers are supposed to represent winding polarity. The "1" wires (H_1 and X_1) represent where the polarity-marking dots would normally be placed.

The similar placement of these dots next to the top ends of the primary and secondary windings tells us that whatever instantaneous voltage polarity is seen across the primary winding will be the same as that across the secondary winding. In other words, the phase shift from primary to secondary will be zero degrees.



On the other hand, if the dots on each winding of the transformer do *not* match up, the phase shift will be 180° between primary and secondary, like this:

Toroids

- **Doughnut-shaped cores** (usually) made of a ferrite material used to wind transformers and inductors.
- **Entire magnetic field** is contained **within** the toroid.

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Reactance

- **Reactance** is the **opposition** to the **flow of Alternating Current (AC).**
- **Reactance** has **no effect** on the flow of **Direct Current (DC).**

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Capacitive Reactance

- **Capacitive Reactance** is the **opposition** to the **flow of AC by capacitance.**
- As the **frequency of the AC increases, Capacitive Reactance decreases.**
- The **Symbol** for **Capacitive Reactance** is $\mathbf{X_C}$.
- $\mathbf{X_C}$ is expressed in **ohms.**
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy **stored in a capacitor** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

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Capacitive Reactance

$$X_C = \frac{1}{2 \pi f C}$$

- Where:

  F = frequency in Hertz

  C = capacitance in Farads

  π = 3.14

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Inductive Reactance

- **Inductive Reactance** is the **opposition** to the **flow of current** in an **AC circuit** caused by an **inductor.**
- As the **frequency increases,** Inductive Reactance **also increases.**
- The **symbol** for **Inductive Reactance** is $\mathbf{X_L}$.
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy **stored in an inductor's magnetic field** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

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Inductive Reactance

$$X_L = 2 \pi f L$$

- Where:

  f = frequency in Hertz

  L = inductance in henrys

  π = 3.14

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Current versus Voltage

- In a simple **resistive** circuit, the **current and voltage are always in phase.**
- The current and voltage are **not in phase in AC circuits** that contain **capacitance and/or inductance.**
- The **current** across a **capacitor leads the voltage by 90 degrees.**
- The **current** across an **inductor lags the voltage by 90 degrees.**

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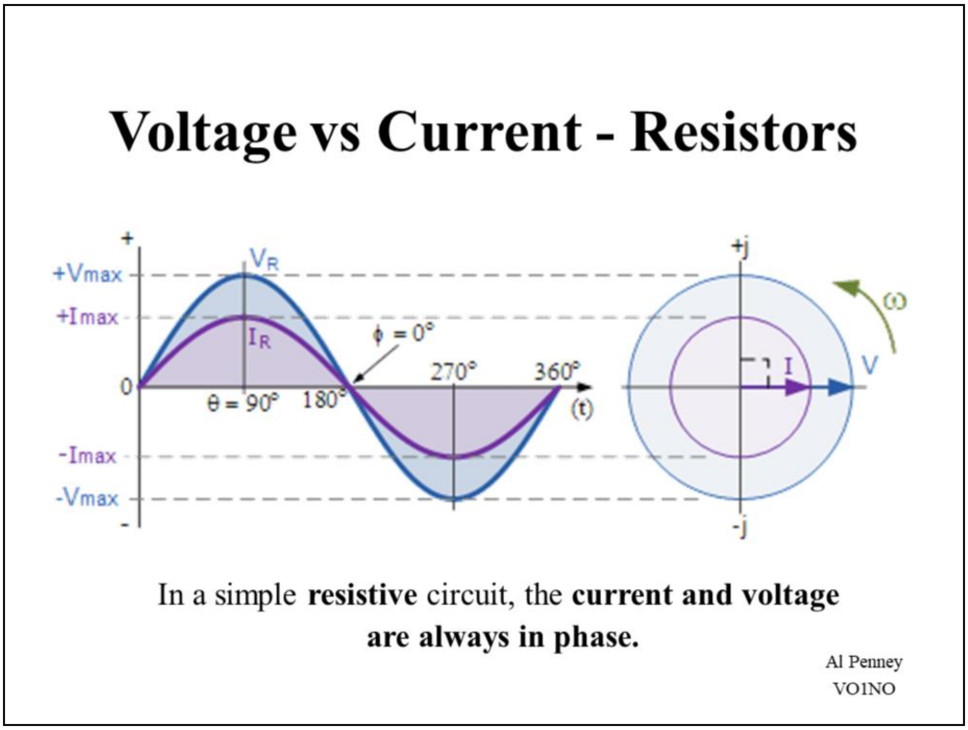
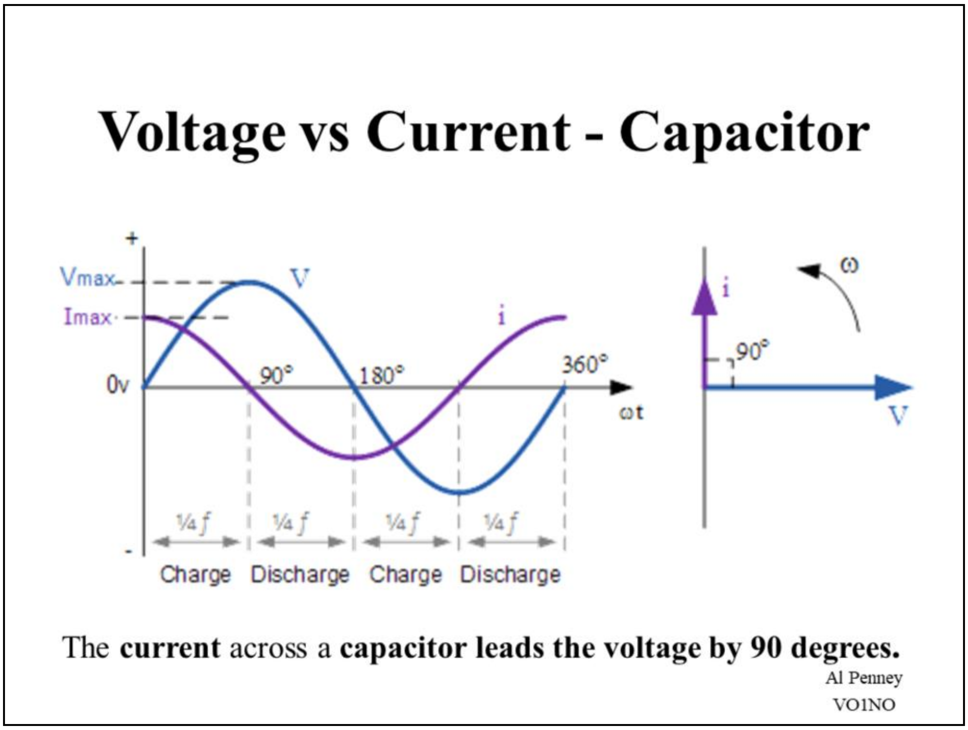


For resistors in AC circuits the direction of the current flowing through them has no effect on the behaviour of the resistor so will rise and fall as the voltage rises and falls. The current and voltage reach maximum, fall through zero and reach minimum at exactly the same time. i.e, they rise and fall simultaneously and are said to be “in-phase” as shown.

We can see that at any point along the horizontal axis that the instantaneous voltage and current are in-phase because the current and the voltage reach their maximum values at the same time, that is their phase angle θ is 0^{o}.



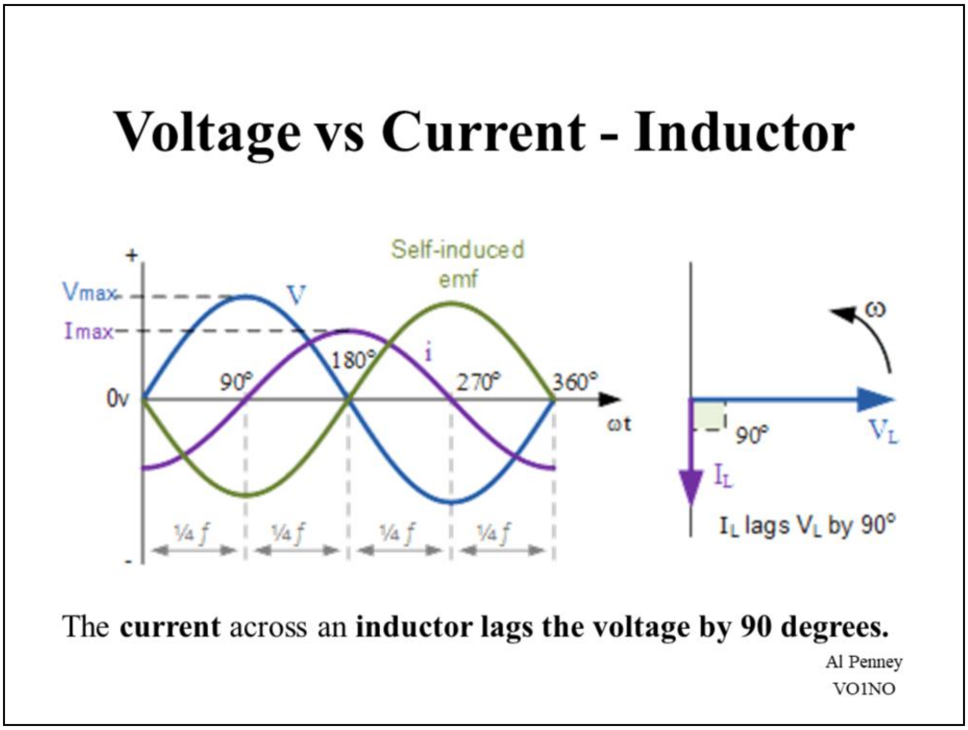
At 0º the rate of change of the supply voltage is increasing in a positive direction resulting in a maximum charging current at that instant in time. As the applied voltage reaches its maximum peak value at 90º for a very brief instant in time the supply voltage is neither increasing or decreasing so there is no current flowing through the circuit.

As the applied voltage begins to decrease to zero at 180º, the slope of the voltage is negative so the capacitor discharges in the negative direction. At the 180º point along the line the rate of change of the voltage is at its maximum again so maximum current flows at that instant and so on.

Then we can say that for capacitors in AC circuits the instantaneous current is at its minimum or zero whenever the applied voltage is at its maximum and likewise the instantaneous value of the current is at its maximum or peak value when the applied voltage is at its minimum or zero.

From the waveform above, we can see that the current is leading the voltage by 1/4 cycle or 90º as shown by the vector diagram. Then we can say that in a purely capacitive circuit the alternating voltage **lags** the current by 90º.

We know that the current flowing through the capacitance in AC circuits is in opposition to the rate of change of the applied voltage but just like resistors, capacitors also offer some form of resistance against the flow of current through the circuit, but with capacitors in AC circuits this AC resistance is known as **Reactance** or more commonly in capacitor circuits, **Capacitive Reactance**, so capacitance in AC circuits suffers from **Capacitive Reactance**.



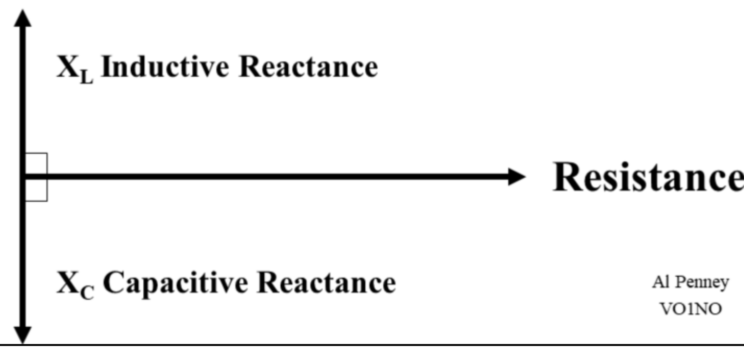
We know that this self-induced emf is directly proportional to the rate of change of the current through the coil and is at its greatest as the supply voltage crosses over from its positive half cycle to its negative half cycle or vice versa at points, 0° and 180° along the sine wave.

Consequently, the minimum rate of change of the voltage occurs when the AC sine wave crosses over at its maximum or minimum peak voltage level. At these positions in the cycle the maximum or minimum currents are flowing through the inductor circuit and this is shown below.

These voltage and current waveforms show that for a purely inductive circuit the current lags the voltage by 90°. Likewise, we can also say that the voltage leads the current by 90°. Either way the general expression is that the current lags as shown in the vector diagram. Here the current vector and the voltage vector are shown displaced by 90°. *The current lags the voltage*.

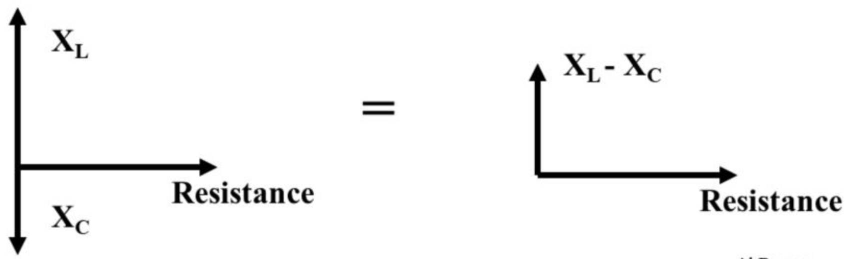
Vector Representation

- When plotted as **vectors, series circuits** containing Inductance, Capacitance and Resistance **(LCR)** can be represented as such:



Inductive vs Capacitive Reactance

- Inductive and Capacitive Reactance **cannot** be **added together** to give an overall reactance.
- In fact, they tend to **cancel each other out.**



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Impedance

- When a circuit contains both **resistance** and **reactance,** the **opposition to the flow of AC** is called **Impedance,** abbreviated **Z.**
- Because **Resistance** and **Reactance** are **not in phase** however, we must use **vectors** to determine the **Impedance,** even if Inductive and Capacitive Reactance have partly **cancelled each other out.**

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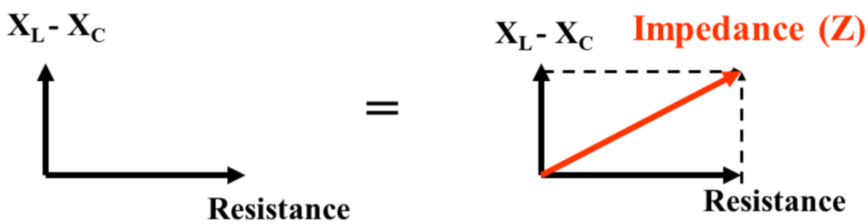
Vector Addition

Through the use of **vector addition,** the Impedance can be determined…



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LCR Circuit Impedance Formula

- Rather than plot vectors every time we need to determine impedance however, we can use a **formula:**

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_c)^2}$$

- Note that because the difference between X_L and X_C is squared, **it doesn't matter** what term is subtracted from what – you can use $\mathbf{X_C - X_L}$ if that is more convenient.

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- X_L = 40 Ohms
- X_C = 130 Ohms

- Z = Sqr Root [$R^2 + (X_C - X_L)^2$]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- X_L = 40 Ohms
- X_C = 130 Ohms

- Z = Sqr Root [$R^2 + (X_C - X_L)^2$]
  = Sqr Root [$(120)^2 + (130 - 40)^2$]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- X_L = 40 Ohms
- X_C = 130 Ohms

- Z = Sqr Root [$R^2 + (X_C - X_L)^2$]
  = Sqr Root [$(120)^2 + (130 - 40)^2$]
  = Sqr Root [14400 + 8100]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
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  = Sqr Root [$(120)^2 + (130 - 40)^2$]
  = Sqr Root [14400 + 8100]
  = Sqr Root [22500]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- X_L = 40 Ohms
- X_C = 130 Ohms

- Z = Sqr Root [$R^2 + (X_C - X_L)^2$]
  = Sqr Root [$(120)^2 + (130 - 40)^2$]
  = Sqr Root [14400 + 8100]
  = Sqr Root [22500]
  = 150 Ohms

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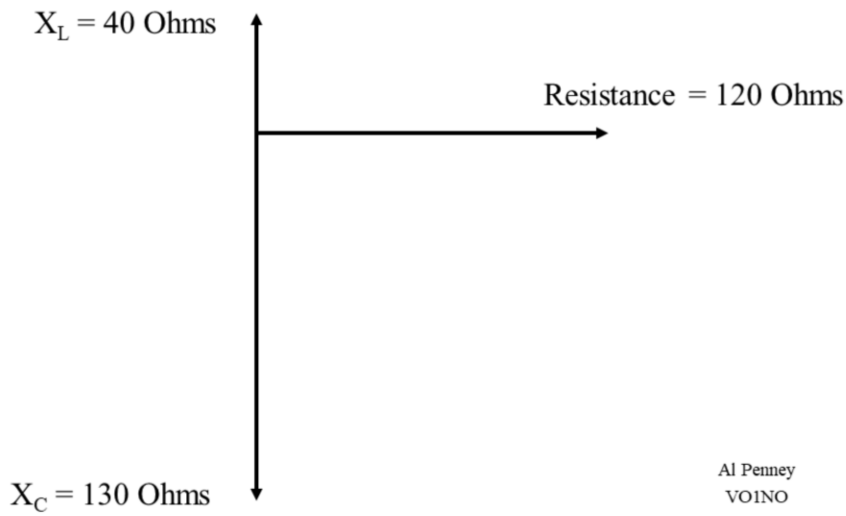
LCR Circuit Impedance Example



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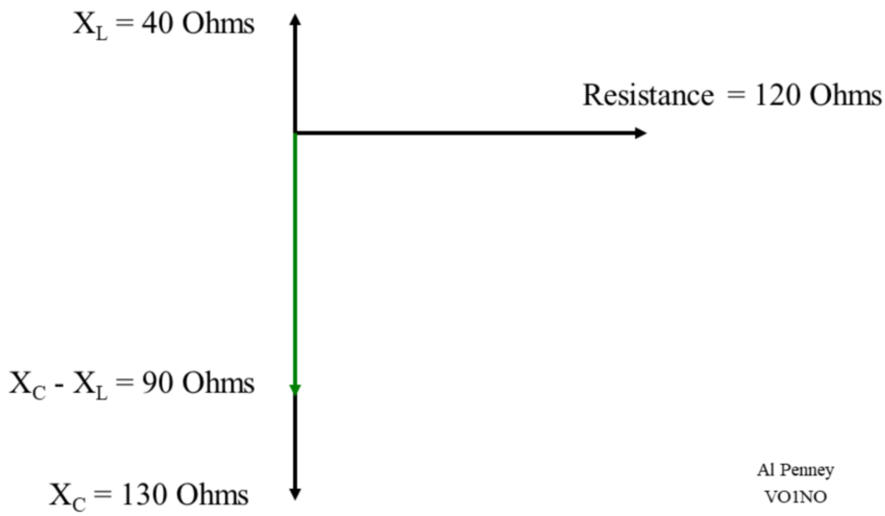
LCR Circuit Impedance Example



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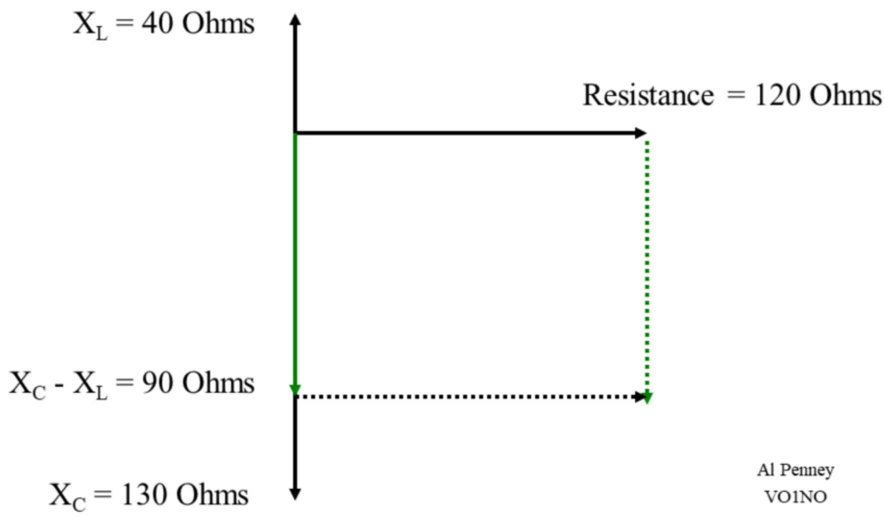
LCR Circuit Impedance Example



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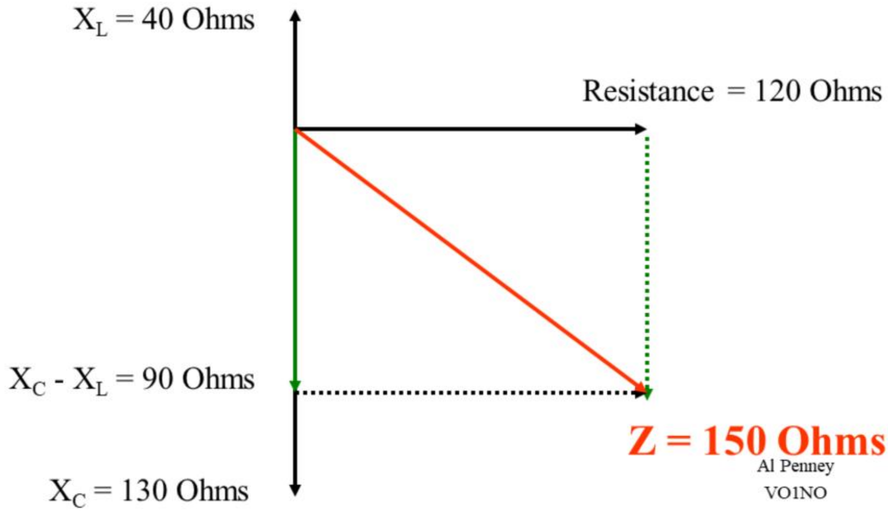
LCR Circuit Impedance Example



Resonance

- In electronic circuits, a special condition exists when **Inductive** and **Capacitive Reactance** are **equal** to each other **($X_L = X_C$).**
- When that happens in Series LCR circuits, **X_L and X_C cancel** each other out, leaving only Resistance to oppose the flow of AC current.
- This condition is know as **Resonance,** and occurs at **only one frequency,** known as the **Resonant Frequency (F_R).**

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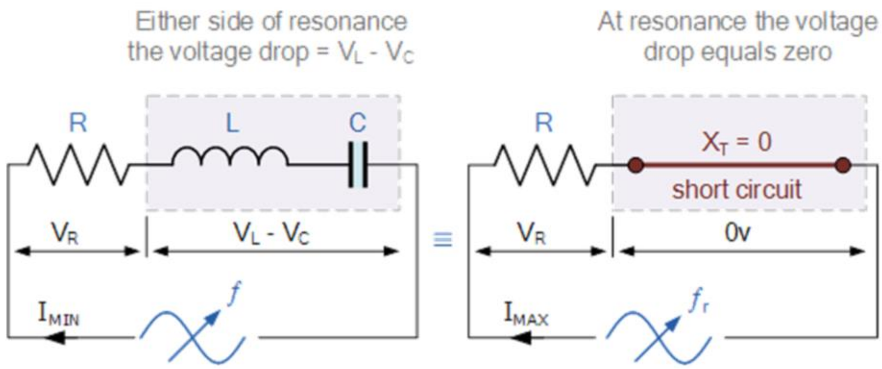
Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is were the two reactance curves cross each other.

Resonance



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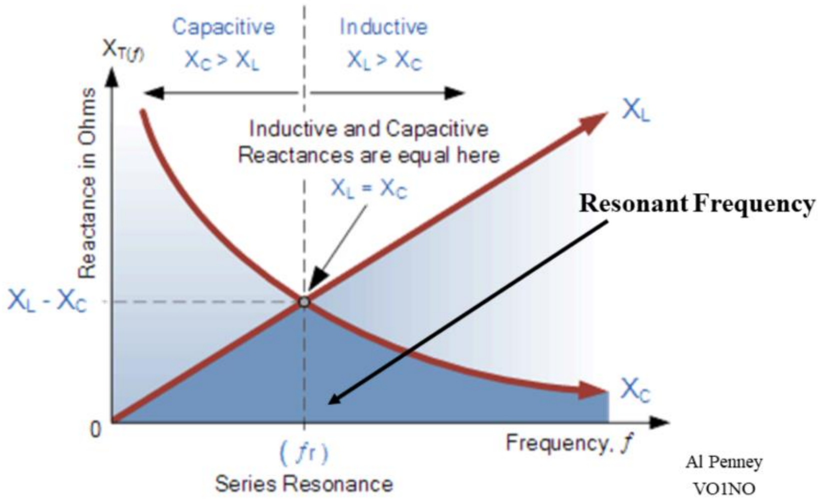
Series Resonant Frequency



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Resonant Frequency

- At Resonance, $\mathbf{X_C = X_L}$ so

$$X_C = \frac{1}{2\pi f C} = X_L = 2\pi f L$$

- With a little mathematical wizardry, we can rearrange that equation to determine the **Resonant Frequency** $\mathbf{F_R}$ as follows…

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Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

- Where:

  F_R = Resonant Frequency in Hertz

  L = Inductance in henrys

  C = Capacitance in Farads

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Resonance is not always a good thing however…

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Tacoma Narrows suspension bridge

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Tuned Circuits

- Circuits containing **Capacitors and Inductors** are often referred to as **Tuned Circuits.**
- They have **many uses** in electronics – every time you tune a radio, you are varying the **resonant frequency** of a **tuned circuit.**

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Series LCR Circuit

- When a **Series LCR** circuit is in **Resonance, current** in that circuit is at its **greatest** (the **Impedance** is at its **lowest**).
- Outside the resonant frequency, the impedance is high, and current therefore low.
- Purpose of a Series LCR circuit is to pass current at the resonant frequency and reject other frequencies.

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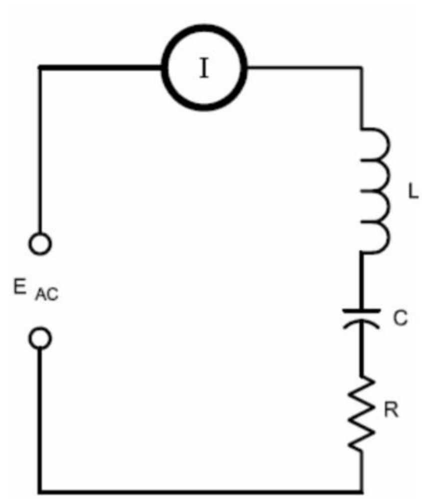
Series LCR Circuit



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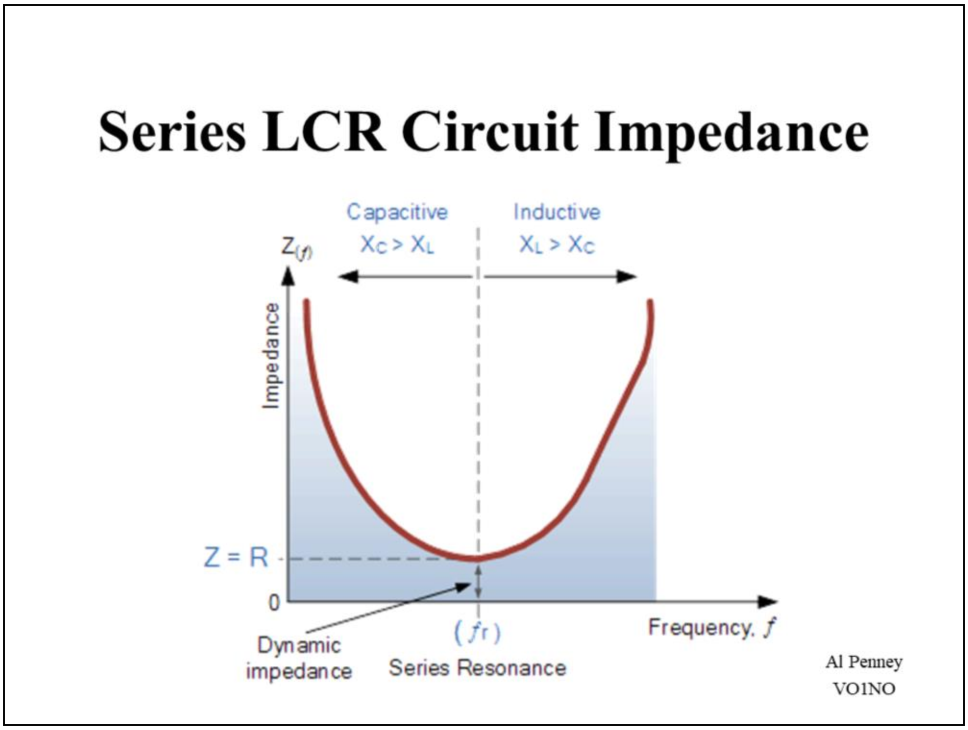
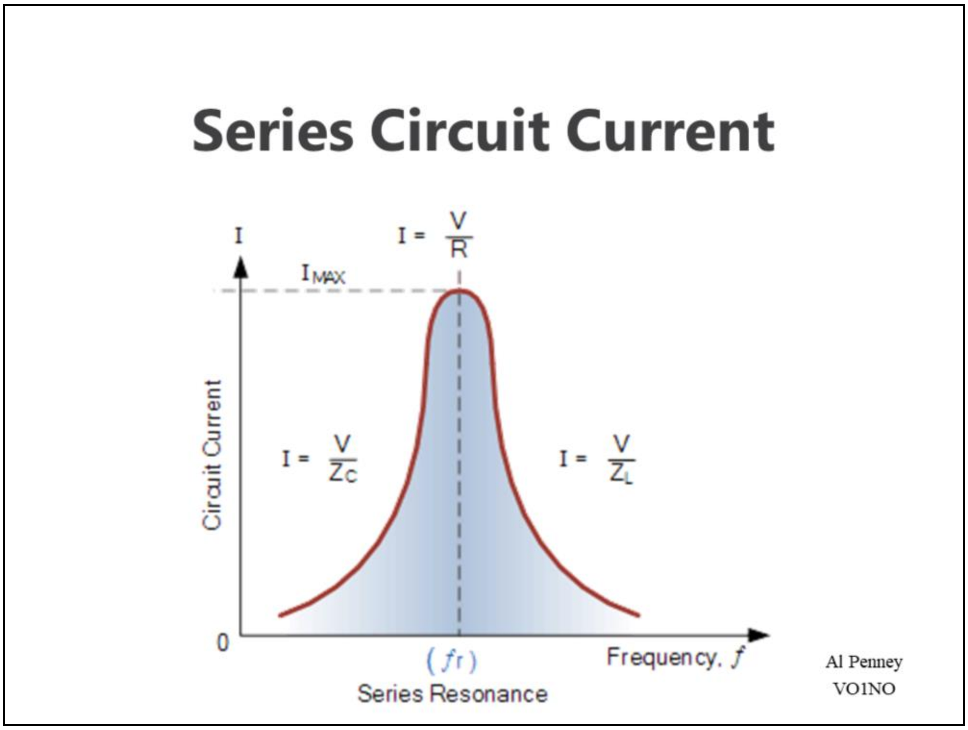


Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L.

You may also note that if the circuits impedance is at its minimum at resonance then consequently, the circuits **admittance** must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the resulting current flowing through the circuit may be dangerously high.

We recall from the previous tutorial about series RLC circuits that the voltage across a series combination is the phasor sum of V_R, V_L and V_C. Then if at resonance the two reactances are equal and cancelling, the two voltages representing V_L and V_C must also be opposite and equal in value thereby cancelling each other out because with pure components the phasor voltages are drawn at $+90^o$ and -90^o respectively.

Then in a **series resonance** circuit as $V_L = -V_C$ the resulting reactive voltages are zero and all the supply voltage is dropped across the resistor. Therefore, $V_R = V_{supply}$ and it is for this reason that series resonance circuits are known as voltage resonance circuits, (as opposed to parallel resonance circuits which are current resonance circuits).



The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite. The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition they cancel each other out.

Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

- Where:

  F_R = Resonant Frequency in Hertz

  L = Inductance in henrys

  C = Capacitance in Farads

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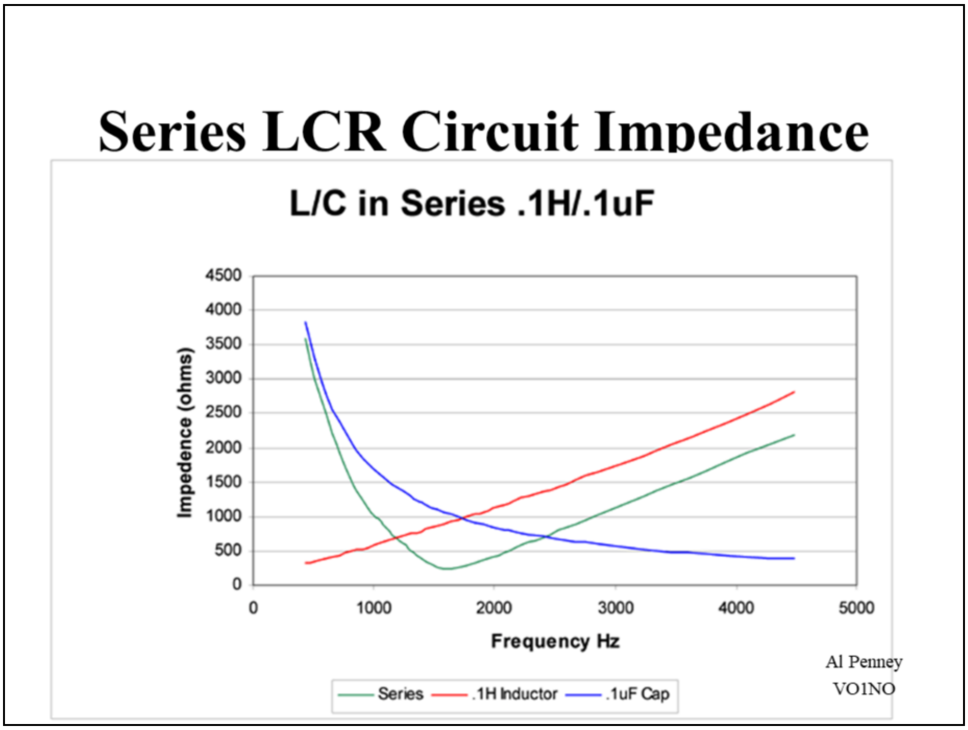
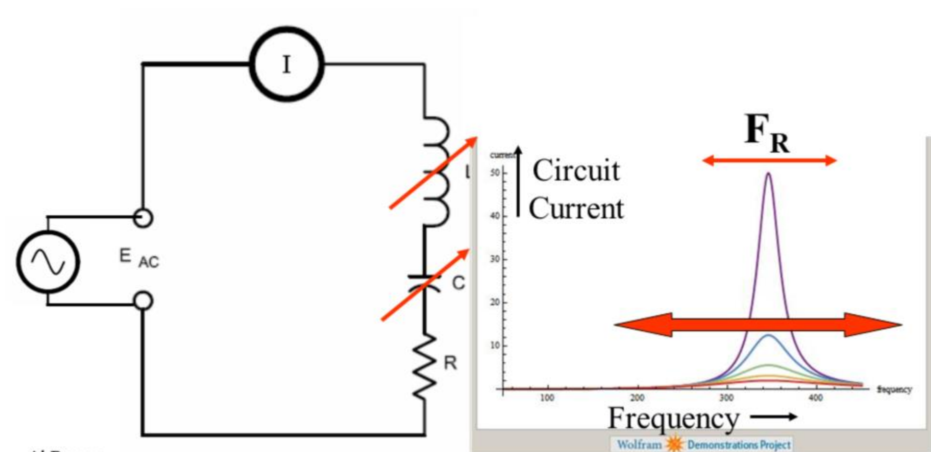


Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L.

Varying Capacitance or Inductance



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Parallel LCR Circuits

- In a **Parallel LCR Circuit**, the **current** is **lowest at Resonance** (the **impedance** is at its **highest).**
- Parallel LCR circuits are used to **reject a specific frequency** while allowing all others to pass.
- Sometimes called a Tank Circuit.

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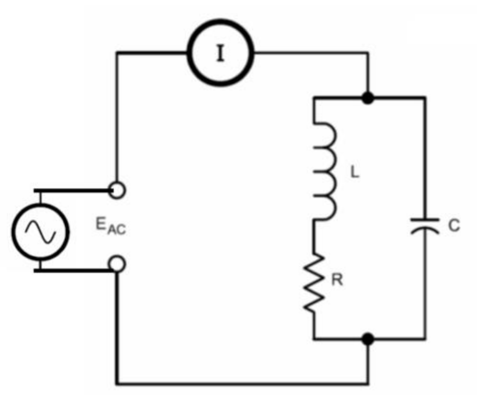
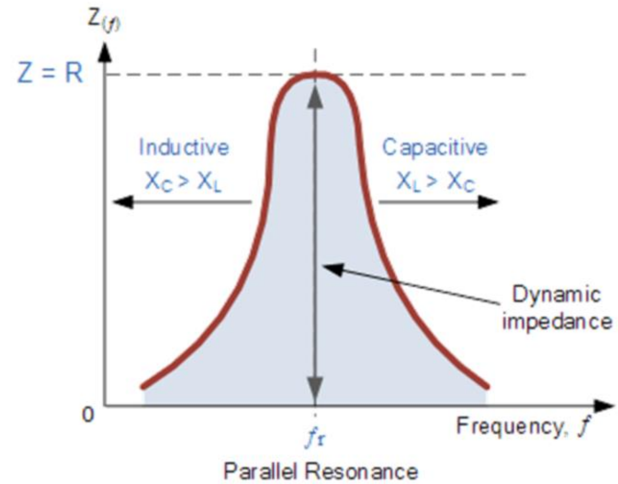
Parallel LCR Circuits



Parallel LCR Circuit Impedance



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Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

- Where:

  F_R = Resonant Frequency in Hertz

  L = Inductance in henrys

  C = Capacitance in Farads

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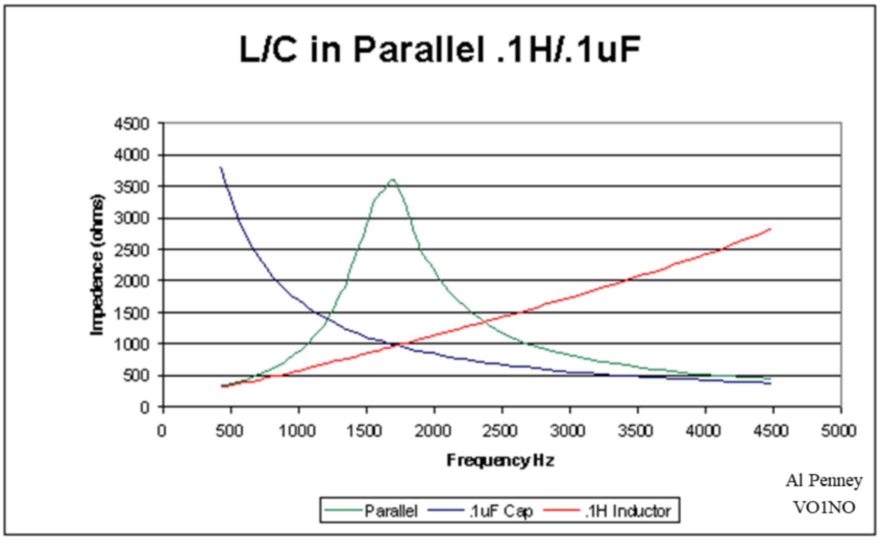
Third time for this slide – **KNOW THIS FORMULA!**

Parallel LCR Circuit Impedance



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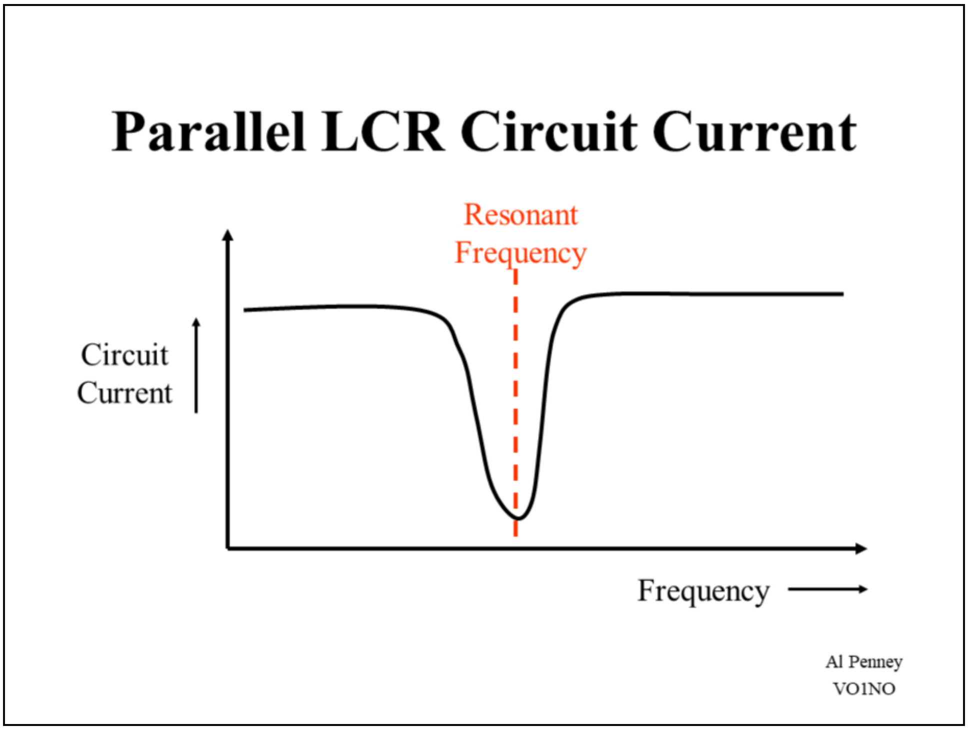

As a parallel resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Rejecter Circuit** because at resonance, the impedance of the circuit is at its maximum thereby suppressing or rejecting the current whose frequency is equal to its resonant frequency. The effect of resonance in a parallel circuit is also called “current resonance”.

The calculations and graphs used above for defining a parallel resonance circuit are similar to those we used for a series circuit. However, the characteristics and graphs drawn for a parallel circuit are exactly opposite to that of series circuits with the parallel circuits maximum and minimum impedance, current and magnification being reversed. Which is why a parallel resonance circuit is also called an **Anti-resonance** circuit.

Circuit Quality

- The **Quality** or “**Q**” of a circuit is a **measure of the “sharpness” of the circuit’s selection of frequencies.**
- High Q circuits are necessary in today’s dense RF environment.
- Typical circuit Qs are 50 to 250.

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Circuit Quality

- In a **resonant circuit,** energy is stored **alternately** in the **electric field of the capacitor,** and then the **magnetic field of the inductor.**
- This causes a **current** to flow **between them.**
- Anything that **removes energy** from this circuit **broadens the range of frequencies** affected by the circuit, but **increases the impedance** at the **resonant frequency.**

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Series LCR Circuit Quality

- The **"Q", or Quality** of a **Series LCR** circuit is defined as the **ratio** of either **X_C or X_L** to the **resistance** in the circuit.
- At resonance $X_C = X_L$
- **"Q" = $X_C / R = X_L / R$**

- Note that most of a LCR circuit's resistance is usually in the inductor windings.

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Parallel LCR Circuit Quality

- The **"Q", or Quality** of a **Parallel LCR** circuit is defined as the **ratio** of the **resistance** to either $\mathbf{X_C}$ **or** $\mathbf{X_L}$ in the circuit.
- **"Q" = $\mathbf{R / X_C = R / X_L}$**

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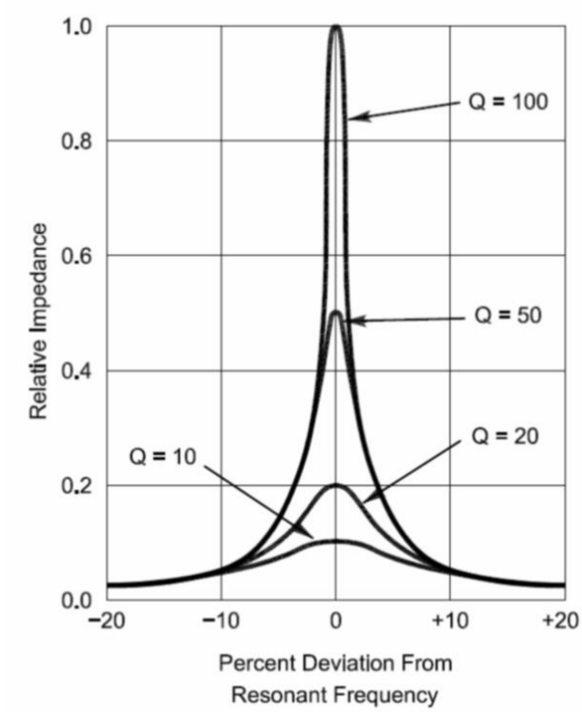

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Inductor Q

- The winding resistance of the inductor is usually the greatest resistance in a circuit.
- The Q of an inductor can be calculated if that resistance is known.
- $\mathbf{Q_L = X_L / R_L = 2\pi FL / R_L}$

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Another Useful Relationship

- If the circuit Q is known bandwidth can be calculated:
- **BW = F / Q**
  - **BW is Bandwidth in Hz**
  - **F is center frequency in Hz**
  - **Q is circuit Q**

- Examples are in Study Guide and Question Bank – **do them!**

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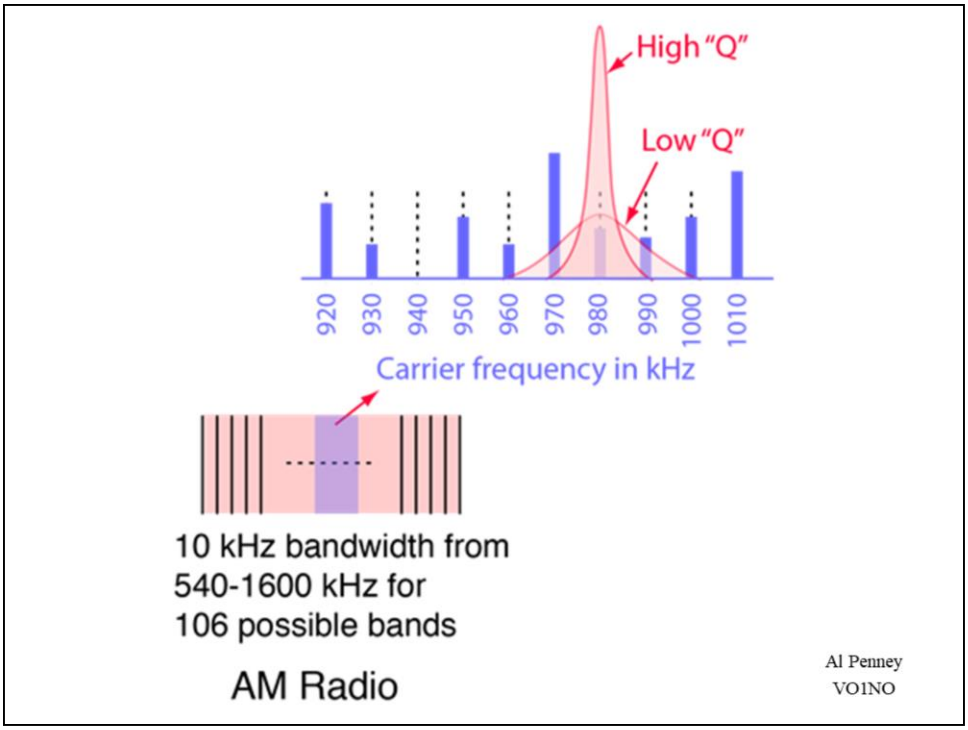
High "Q"
Low "Q"
920
930
940
950
960
970
980
990
1000
1010
Carrier frequency in kHz
10 kHz bandwidth from
540-1600 kHz for
106 possible bands
AM Radio
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Skin Effect

- Tendency of AC to become distributed within a conductor such that the current density is largest near the surface of the conductor, and decreases with greater depths in the conductor.
- The electric current flows mainly at the "skin" of the conductor, between the outer surface and a level called the **skin depth**.
- The skin effect causes the effective resistance to increase at higher frequencies where the skin depth is smaller, thus reducing the effective cross-section of the conductor.

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Skin Effect

- Skin effect is due to opposing eddy currents induced by the changing magnetic field resulting from the alternating current.
- At 60 Hz in copper, the skin depth is ~ 8.5 mm.
- At high frequencies the skin depth becomes much smaller.
- Increased AC resistance due to the skin effect can be mitigated by using specially woven litz wire.
- Because the interior of a large conductor carries so little of the current, tubular conductors such as pipe can be used to save weight and cost.

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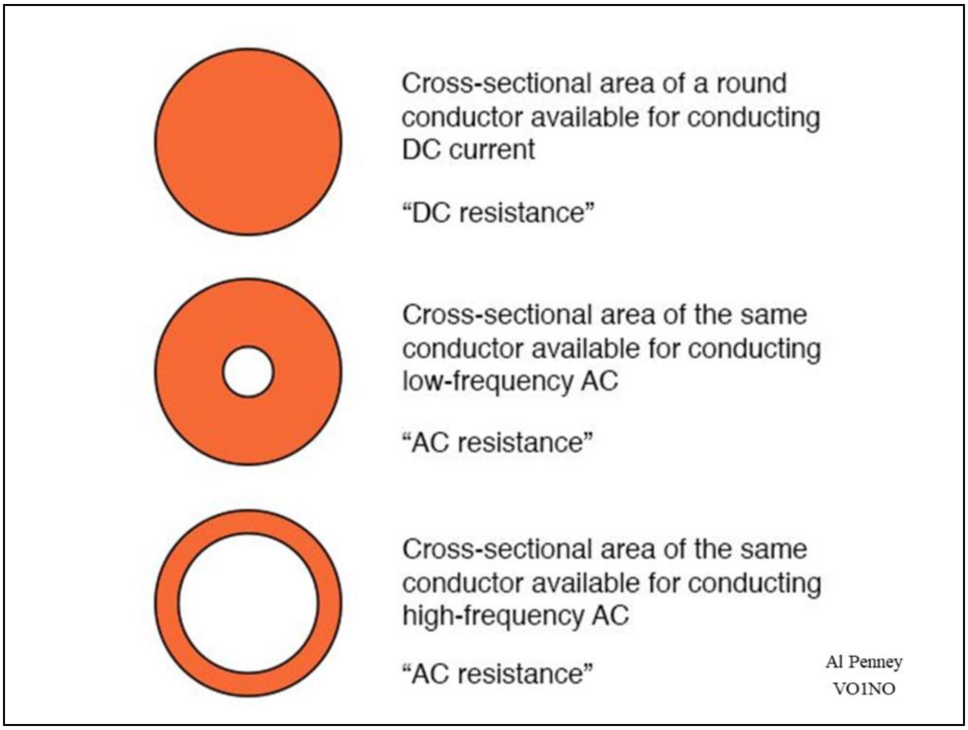
Cross-sectional area of a round conductor available for conducting DC current
"DC resistance"
Cross-sectional area of the same conductor available for conducting low-frequency AC
"AC resistance"
Cross-sectional area of the same conductor available for conducting high-frequency AC
"AC resistance"
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RC Time Constant

- It takes a finite amount of time for a capacitor to charge when a voltage is applied.
- Resistance in the circuit increases this time.
- This delay is measured in units called the Time Constant, abbreviated τ (Tau).
- **τ = R x C, where τ is in seconds, R in Ohms, and C in Farads.**

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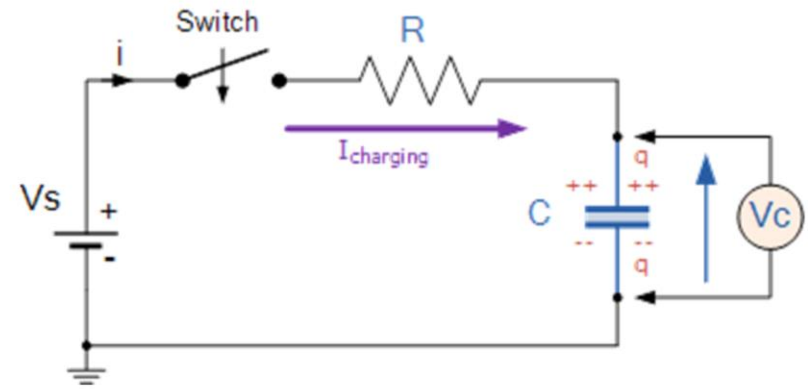

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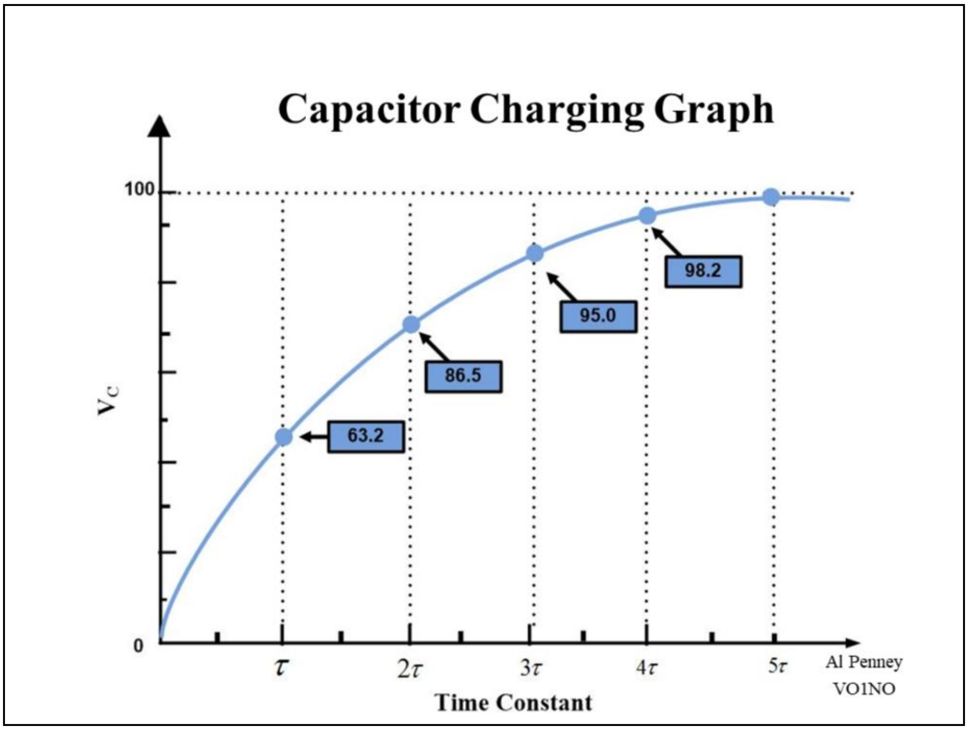
Capacitor Charging Graph
100
0
V_C
63.2
86.5
95.0
98.2
τ
2τ
3τ
4τ
5τ
Time Constant
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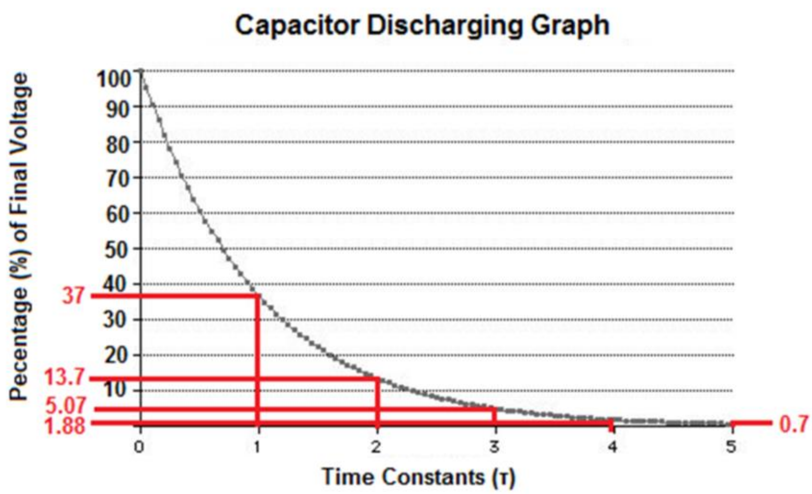

- **NOTE**: A capacitor is considered fully charged or discharged after **5 time constants**.

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RL Time Constant

- Just as with voltage in an RC circuit, the current in an RL circuit takes a finite time to build up or decrease.
- The same concepts of time constant etc. apply to RL circuits, but to current.
- Note that R may only be the inductor's own resistance.
- **$\tau = L / R$, τ in seconds, L in henrys, R in ohms.**

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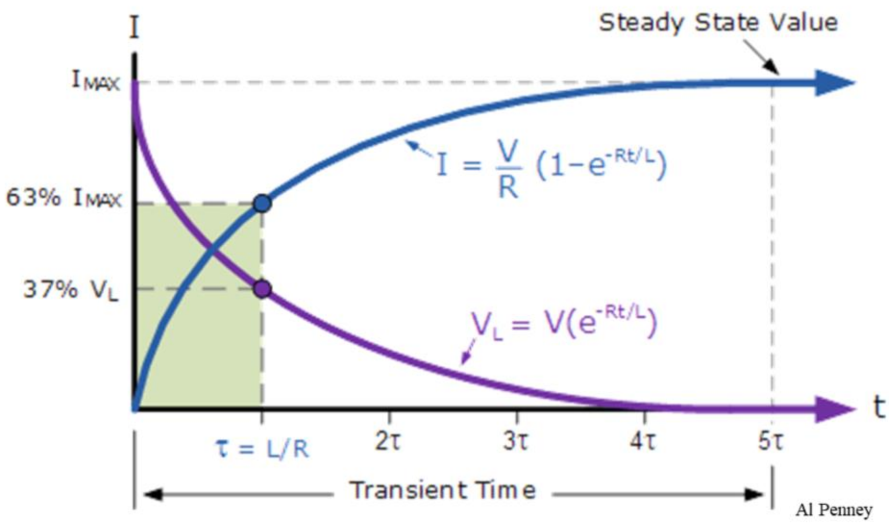

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Questions?

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What is the resonant frequency of a series RLC circuit, if R is 47 ohms, L is 4 microhenrys and C is 20 picofarads? □ 19.9 MHz □ 17.8 MHz □ 19.9 kHz □ 17.8 kHz

< 17.8 MHz >

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This problem involves using the formula that you will find in Section 1.14 of the Advanced Study Guide for calculating the resonant frequency of a series RLC circuit. The factors in the formula are: fR = resonant frequency in Hz, pi = 3.14, L = inductance in henrys and C = capacitance in farads. You also have to be comfortable with changing units as you will see as we show you how to answer the question. Since L is in microhenrys, we have to convert it to henrys. 4 microhenrys = 0.000 004 henrys or 4.0 x 10-6 if we use scientific notation. A similar calculation has to be done to change picofarads to farads. 20 picofarads = 0.000 000 000 020 farads or 20 x 10-12 farads if we use scientific notation. This problem is best solved in steps unless you are really comfortable with your calculator. We are assuming that you have a scientific calculator. Multiply 0.000 004 by 0.000 000 000 020 and then hit the SQRT (square root) key. Your calculator may show the symbol √ for "square root" – this symbol is used in the formula. Given that your calculator may not handle this you might be better off to use the scientific notation function of your calculator to do this calculation. For this question L = 4.0 x 10-06 so you would key in 4.0 first. Then tap the EXP key and enter - 6. Some calculators may automatically enter a 0 in front so your display may appear as -06. Now tap the multiply key and enter the value for C, 20 x 10-12, in a similar fashion. Now tap the SQRT key. Regardless of which method you employed DO NOT clear your calculator. This value will be 8.994 x 10-9 to three decimal places. Don't worry about the numbers beyond three decimal places; we will leave them in and deal with them later. Multiply the value the value above by 2 and then by pi, 3.14, and tap the = key. This will yield a value of 5.617 x 10-8 to three decimal places. Again, don't worry about the numbers beyond three decimal places; we will leave them in and deal with them later. 21 Now comes the easy part. Look for the reciprocal key on your calculator; it is usually labeled 1/x. Tap this key and up pops 17 803 088. DO NOT clear your calculator. Our calculated value is in Hz and our answers are given in MHz and kHz. We convert Hz to MHz by dividing 17 803 088 by 1 000 000, yielding 17.803088 MHz. Converting to kHz we get 17 803.088 kHz. The correct answer is shown to two decimal places so our calculated value looks a lot like the answer when we round off 17.803088 MHz to 17.8 MHz.

What is the value of capacitance (C) in a series RLC circuit, if the circuit resonant frequency is 14.25 MHz and L is 2.84 microhenrys? 2.2 microfarads 44 microfarads 44 picofarads 2.2 picofarads

< 44 picofarads >

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This is a very practical problem. Imagine that you are trying to construct an RLC circuit resonant at a specific frequency. Your junk box yields a capacitor or inductor with a fixed value. What is the value of the other component you will have to procure? This problem involves using the formula you will find in Section 1.14 in the Advanced Study Guide for 23 calculating the resonant frequency of a parallel RLC circuit. The factors in the formula are: fR = resonant frequency in Hz, pi = 3.14, L = inductance in henrys and C = capacitance in farads. You also have to be comfortable with changing units as you will see as we show you how to answer the question. However, this problem has “thrown you a curve”. Unlike the other resonance calculation questions this one gives you the resonant frequency and the value of the inductance. You have to find the capacitance. We will have to re-write the equation as follows: 1 퓟퓡2휇 = √퓛퓒 On the left all our values are “knowns”. We can use these to find the value of SQRT LC. Once we know this we can use it and the value of L to find the value of C. Don’t despair – all will be revealed. Before we start crunching numbers we need to do some unit conversions. fR = 14.25 MHz = 14 250 000 Hz = 14.25 x 106 Hz. C = 44 picofarads = 0.000 000 000 044 farads = 44 x 10-12 farads. We want to find the value of 1 퓟퓡2휇 . The simplest route is to use scientific notation. 1/((14.25 x 106)(2)(3.14)) = 11.17 x 10-9 We have just calculated the value of SQRT (LC). We now want to find the value of LC. To do this we have to square 11.17 x 10-9 (multiply it by itself). You can enter the value into your calculator and look for the key that squares any value or you can simply enter it again and tap the MULTIPLY key. Regardless of the method you employ the result will be 1.248 x 10-16 to three decimal places. We now know that LC = 1.248 x 10-16 and we know the value of L. So the task is to find the value of C, which will be 1.248 x 10-16 / L. Plugging in all the numbers we find 1.248 x 10-16 / 28.4 x 10-6 = 43.94 x 10- 12 farads. The size of the exponent suggests that our answer will be in picofarads, so we convert farads to picofarads. If we round this off to 44 picofarads we find we have the value needed.

What is the resonant frequency of a parallel RLC circuit if R is 4.7 kilohms, L is 1 microhenry and C is 10 picofarads? □ 15.9 kHz □ 50.3 MHz □ 50.3 kHz □ 15.9 MHz

< 50.3 MHz >

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This problem involves using the formula you will find in Section 1.14 in the Advanced Study Guide for 24 calculating the resonant frequency of a parallel RLC circuit. The factors in the formula are: fR = resonant frequency in Hz, pi = 3.14, L = inductance in henrys and C = capacitance in farads. You also have to be comfortable with changing units as you will see as we show you how to solve the question. Since L is in microhenrys, we have to convert it to henrys. 1 microhenry = 0.000 001 henrys or 1.0 x 10-6 if we use scientific notation. A similar calculation has to be done to change picofarads to farads. 10 picofarads = 0.000 000 000 010 farads or 10 x 10-12. This problem is best solved in steps unless you are really comfortable with your calculator. We are assuming that you have a scientific calculator. Multiply 0.000 001 by 0.000 000 000 010 and then hit the SQRT (square root) key. Your calculator may show the symbol for "square root" – this symbol is used in the formula. Given that your calculator may not handle this you might be better off to use the scientific notation function of your calculator to do this calculation. For this question L = 1.0 x 10-06. For L you would key in 1.0 first. Then tap the EXP key and enter -6. Some calculators may automatically enter a 0 in front so your display may appear as -06. Now tap the multiply key and enter the value for C, 10 x 10-12. Now tap the SQRT key. Regardless of which method you employed DO NOT clear your calculator. This value will be 3.163 x 10-9 to three decimal places. Don't worry about the numbers beyond three decimal places; we will leave them in and deal with them later. Multiply the value the value above by 2 and then by pi, 3.14, and tap the = key. This will yield a value of 19.85 x 10-9 to three decimal places. Again, don't worry about the numbers beyond three decimal places; we will leave them in and deal with them later. Now comes the easy part. Look for the reciprocal key on your calculator; it is usually labelled 1/x. Tap this key and up pops 50 354 739. DO NOT clear your calculator. Our calculated value is in Hz and our answers are given in MHz and kHz. We convert Hz to MHz by dividing by 1 000 000, yielding 50.354739 MHz. Converting to kHz we get 50354.739 kHz. The correct answer is shown to one decimal place so our calculated value looks a lot like the answer in MHz when we round off 50.354939 MHz to 50.4 MHz.

What is the value of inductance (L) in a parallel RLC circuit, if the resonant frequency is 14.25 MHz and C is 44 picofarads? 253.8 millihenrys 3.9 millihenrys 0.353 microhenry 2.8 microhenrys

< 2.8 microhenrys >

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This is a very practical problem. Imagine that you are trying to construct an RLC circuit resonant at a specific frequency. Your junk box yields a capacitor or inductor with a fixed value. What is the value of the other component you will have to procure? This problem involves using the formula you will find in Section 1.14 in the Advanced Study Guide for 28 calculating the resonant frequency of a parallel RLC circuit. The factors in the formula are: fR = resonant frequency in Hz, pi = 3.14, L = inductance in henrys and C = capacitance in farads. You also have to be comfortable with changing units as you will see as we show you how to answer the question. However, this problem has "thrown you a curve". Unlike the other resonance calculation questions this one gives you the resonant frequency and the value of the capacitance. You have to find the inductance. We will have to re-write the equation as follows: 1 푓핀2휂 = √팰팺 On the left all our values are "knowns". We can use these to find the value of SQRT LC. Once we know this we can use it and the value of C to find the value of L. Don't despair – all will be revealed. Before we start crunching numbers we need to do some unit conversions. fR = 14.25 MHz = 14 250 000 Hz = 14.25 x 106 Hz. C = 44 picofarads = 0.000 000 000 044 farads = 44 x 10-12 farads . We want to find the value of 1 푓핀2휂 . The simplest route is to use scientific notation. 1/((14.25 x 106)(2)(3.14)) = 11.17 x 10-9 We have just calculated the value of SQRT (LC). We now want to find the value of LC. To do this we have to square 11.17 x 10-9 (multiply it by itself). You can enter the value into your calculator and look for the key that squares any value or you can simply enter it again and tap the MULTIPLY key. Regardless of the method you employ the result will be 1.248 x 10-16 to three decimal places. We now know that LC = 1.248 x 10-16 and we know the value of C. So the task is to find the value of L, which will be will be 1.248 x 10-16 /C. Plugging in all the numbers we find 1.248 x 10-16 / 44 x 10-12 = 2.836 x 10-6 henrys. The size of exponent, 10-6, suggests that our final answer should be expressed in microhenrys so we convert 2.836 x 10-6 henrys to microhenrys by multiplying by 1 000 000, 1 x 10-6. This gives us a value of 2.836. To one place of decimals this rounds down to 2.8 microhenrys, the same.

What is the Q of a parallel RLC circuit, if it is resonant at 14.128 MHz, L is 2.7 microhenrys and R is 18 kilohms? □ 7.51 □ 0.013 □ 71.5 □ 75.1

< 75.1 >

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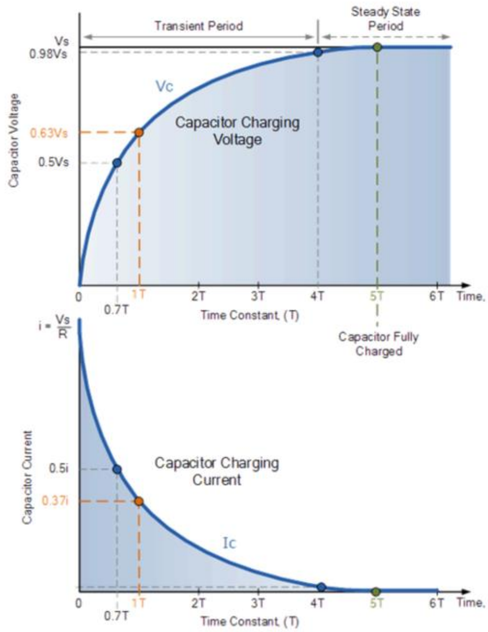
Since it is a parallel circuit use the following formula: $Q = R/2\pi fL$. Ensure that kilohms are converted to ohms, f is converted from MHz to Hz, and microhenries are converted to henrys.



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$$\tau \equiv R \times C$$

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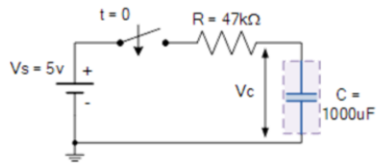

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The time constant, τ is found using the formula T = R x C in seconds.

Therefore the time constant τ is given as: T = R x C = 47k x 1000uF = 47 Secs

a) **What value will be the voltage across the capacitor at 0.7 time constants?**

At 0.7 time constants (0.7T) Vc = 0.5Vs. Therefore, Vc = 0.5 x 5V = 2.5V

b) **What value will be the voltage across the capacitor at 1 time constant?**

At 1 time constant (1T) Vc = 0.63Vs. Therefore, Vc = 0.63 x 5V = 3.15V

c) **How long will it take to "fully charge" the capacitor?**

The capacitor will be fully charged at 5 time constants.

1 time constant (1T) = 47 seconds, (from above). Therefore, 5T = 5 x 47 = 235 secs

d) **The voltage across the Capacitor after 100 seconds?**

The voltage formula is given as $Vc = V(1 – e^{(-t/RC)})$ so this becomes: $Vc = 5(1 – e^{(-100/47)})$

Where: V = 5 volts, t = 100 seconds, and RC = 47 seconds from above.

Therefore, $V_c = 5(1 - e^{(-100/47)}) = 5(1 - e^{-2.1277}) = 5(1 - 0.1191) =$ <u>4.4 volts</u>