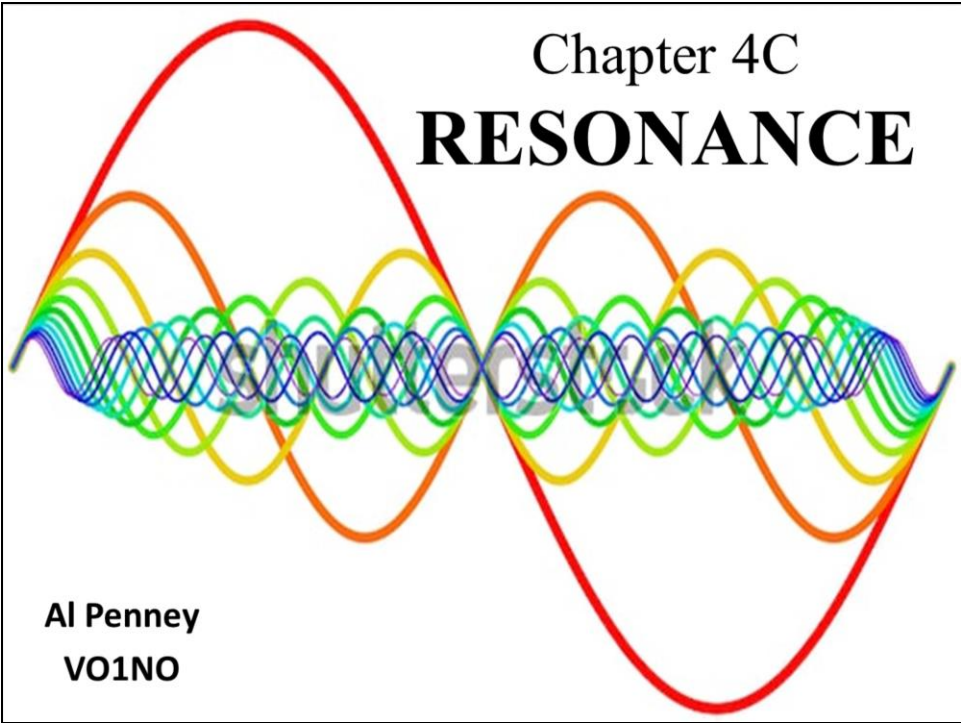


Chapter 4C
RESONANCE



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Objectives

- On completion, you should be able to:
 - Define **Impedance** and **Resonance**;
 - Perform **basic calculations** involving **Impedance** and **Resonant Frequency**;
 - Explain how **Series** and **Parallel LCR Circuits** work; and
 - Explain how Inductors and Capacitors are used in **Filter circuits**.

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Quick Review - Reactance

- **Reactance** is the **opposition** to the **flow of Alternating Current (AC)** caused by **inductance or capacitance**.
- **Larger reactance leads to smaller currents for the same voltage applied.**
- **Reactance has no effect** on the flow of **Direct Current (DC)**.

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In electric and electronic systems, **reactance** is the opposition of a **circuit element** to the flow of alternating **current** due to that element's **inductance** or **capacitance**. Larger reactance leads to smaller currents for the same voltage applied. Reactance is similar to **electric resistance**, but it differs in several respects.

Capacitive Reactance

- **Capacitive Reactance** is the **opposition** to the **flow of AC** by **capacitance**.
- As the **frequency of the AC increases**, **Capacitive Reactance decreases**.
- The **Symbol for Capacitive Reactance** is X_C .
- X_C is expressed in **ohms**.
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy stored in a **capacitor** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

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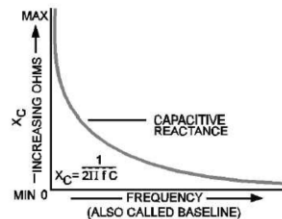
Capacitive reactance (symbol X_C) is a measure of a **capacitor's** opposition to AC (alternating current). Like resistance it is measured in ohms, but **reactance** is more complex than resistance because its value depends on the frequency (f) of the signal passing through the **capacitor**.

Reactance is also inversely proportional to the value of capacitance (C), i.e. the value of X_C at any frequency will be less in larger capacitors than in smaller ones. All capacitors have infinitely high values of reactance at 0Hz, but in large capacitors, the reactance falls to a low level at much lower frequencies than in smaller capacitors. Hence, larger capacitors are preferred in low frequency applications.

Capacitive Reactance

$$X_C = \frac{1}{2\pi f C}$$

- Where:
F = frequency in Hertz
C = capacitance in Farads
 π = 3.14



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At low frequencies a capacitor is an **open circuit** so no **current** flows in the dielectric.

A **DC** voltage applied across a capacitor causes positive **charge** to accumulate on one side and negative **charge** to accumulate on the other side; the **electric field** due to the accumulated charge is the source of the opposition to the current. When the **potential** associated with the charge exactly balances the applied voltage, the current goes to zero.

Driven by an AC supply (ideal AC current source), a capacitor will only accumulate a limited amount of charge before the potential difference changes polarity and the charge is returned to the source. The higher the frequency, the less charge will accumulate and the smaller the opposition to the current.

Inductive Reactance

- **Inductive Reactance** is the **opposition** to the **flow of current** in an **AC circuit** caused by an **inductor**.
- As the **frequency increases**, Inductive Reactance **also increases**.
- The **symbol** for **Inductive Reactance** is **X_L** .
- Even though it is expressed in ohms, **power is not dissipated by Reactance!** Energy stored in an **inductor's magnetic field** during **one part of the AC cycle** is simply **returned to the circuit** during the **next part of the cycle!**

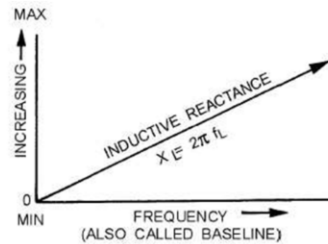
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In [electromagnetism](#) and [electronics](#), **inductance** is the tendency of an [electrical conductor](#) to oppose a change in the [electric current](#) flowing through it. The flow of electric current through a conductor creates a [magnetic field](#) around the conductor, whose strength depends on the magnitude of the current. A change in current causes a change in the magnetic field. From [Faraday's law of induction](#), any change in magnetic field through a circuit induces an [electromotive force](#) (EMF) ([voltage](#)) in the conductors; this is known as [electromagnetic induction](#). So the changing current induces a voltage in the conductor. This induced voltage is in a direction which tends to oppose the change in current (as stated by [Lenz's law](#)), so it is called a *back EMF*. Due to this back EMF, a conductor's inductance opposes any increase or decrease in electric current through it.

Inductive Reactance

$$X_L = 2 \pi f L$$

- Where:
f = frequency in Hertz
L = inductance in henrys
 $\pi = 3.14$



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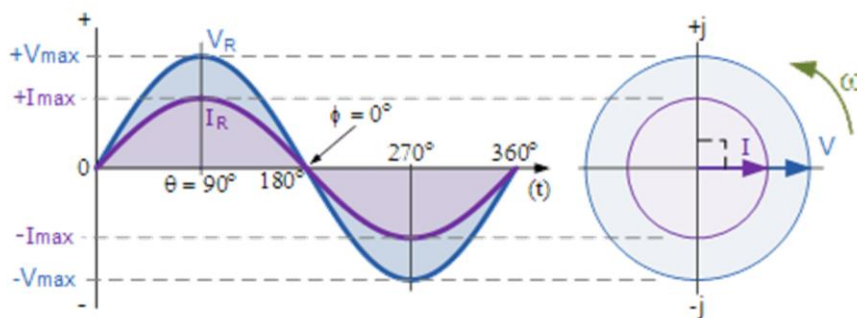
The counter-emf is the source of the opposition to current flow. A constant **direct current** has a zero rate-of-change, and sees an inductor as a **short-circuit** (it is typically made from a material with a low **resistivity**). An **alternating current** has a time-averaged rate-of-change that is proportional to frequency, this causes the increase in inductive reactance with frequency.

Current versus Voltage

- In a simple **resistive** circuit, the **current and voltage are always in phase.**
- The current and voltage are **not in phase in AC circuits** that contain **capacitance and/or inductance.**
- The **current** across a **capacitor leads the voltage by 90 degrees.**
- The **current** across an **inductor lags the voltage by 90 degrees.**

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Voltage vs Current - Resistors



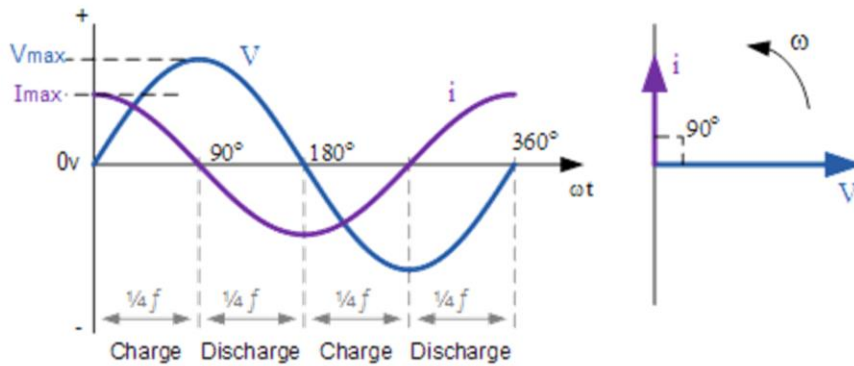
In a simple **resistive** circuit, the **current and voltage** are always **in phase**.

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For resistors in AC circuits the direction of the current flowing through them has no effect on the behaviour of the resistor so will rise and fall as the voltage rises and falls. The current and voltage reach maximum, fall through zero and reach minimum at exactly the same time. i.e, they rise and fall simultaneously and are said to be “in-phase” as shown.

We can see that at any point along the horizontal axis that the instantaneous voltage and current are in-phase because the current and the voltage reach their maximum values at the same time, that is their phase angle θ is 0° .

Voltage vs Current - Capacitor



The **current** across a **capacitor** leads the **voltage** by **90 degrees**.

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At 0° the rate of change of the supply voltage is increasing in a positive direction resulting in a maximum charging current at that instant in time. As the applied voltage reaches its maximum peak value at 90° for a very brief instant in time the supply voltage is neither increasing or decreasing so there is no current flowing through the circuit.

As the applied voltage begins to decrease to zero at 180° , the slope of the voltage is negative so the capacitor discharges in the negative direction. At the 180° point along the line the rate of change of the voltage is at its maximum again so maximum current flows at that instant and so on.

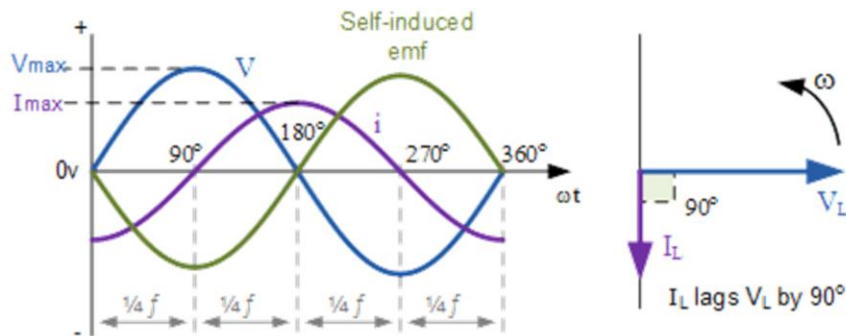
Then we can say that for capacitors in AC circuits the instantaneous current is at its minimum or zero whenever the applied voltage is at its maximum and likewise the instantaneous value of the current is at its maximum or peak value when the applied voltage is at its minimum or zero.

From the waveform above, we can see that the current is leading the voltage by $1/4$ cycle or 90° as shown by the vector diagram. Then we can say that in a purely capacitive circuit the alternating voltage **lags** the current by 90° .

We know that the current flowing through the capacitance in AC

circuits is in opposition to the rate of change of the applied voltage but just like resistors, capacitors also offer some form of opposition against the flow of current through the circuit, but with capacitors in AC circuits this AC resistance is known as **Reactance** or more commonly in capacitor circuits, **Capacitive Reactance**, so capacitance in AC circuits suffers from **Capacitive Reactance**.

Voltage vs Current - Inductor



The **current** across an **inductor** lags the **voltage** by **90 degrees**.

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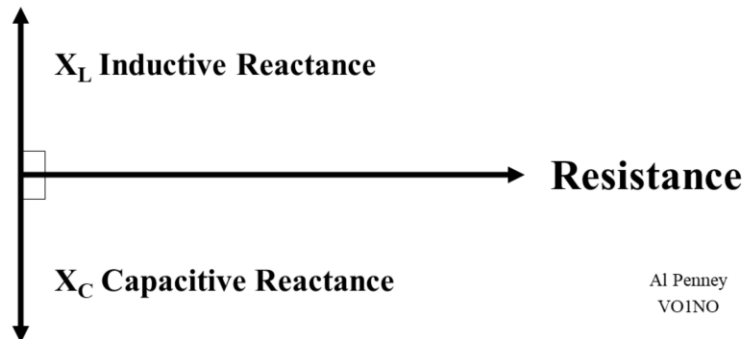
We know that this self-induced emf is directly proportional to the rate of change of the current through the coil and is at its greatest as the supply voltage crosses over from its positive half cycle to its negative half cycle or vice versa at points, 0° and 180° along the sine wave.

Consequently, the minimum rate of change of the voltage occurs when the AC sine wave crosses over at its maximum or minimum peak voltage level. At these positions in the cycle the maximum or minimum currents are flowing through the inductor circuit and this is shown below.

These voltage and current waveforms show that for a purely inductive circuit the current lags the voltage by 90° . Likewise, we can also say that the voltage leads the current by 90° . Either way the general expression is that the current lags as shown in the vector diagram. Here the current vector and the voltage vector are shown displaced by 90° . *The current lags the voltage.*

Vector Representation

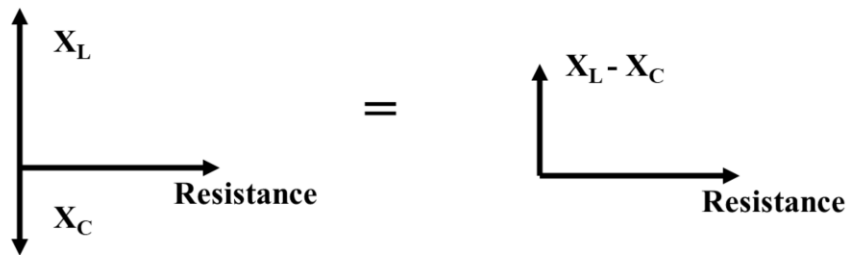
- When plotted as **vectors**, **series circuits** containing Inductance, Capacitance and Resistance (**LCR**) can be represented as such:



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Inductive vs Capacitive Reactance

- Inductive and Capacitive Reactance **cannot** be **added together** to give an overall reactance.
- In fact, they tend to **cancel each other out**.



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Impedance

- When a circuit contains both **resistance** and **reactance**, the **opposition to the flow of AC** is called **Impedance**, abbreviated **Z**.
- Because **Resistance** and **Reactance** are **not in phase** however, we must use **vectors** to determine the **Impedance**, even if Inductive and Capacitive Reactance have partly **cancelled each other out**.

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Impedance - the effective opposition to the flow of alternating current in an electric circuit or component, arising from the combined effects of ohmic resistance and reactance.

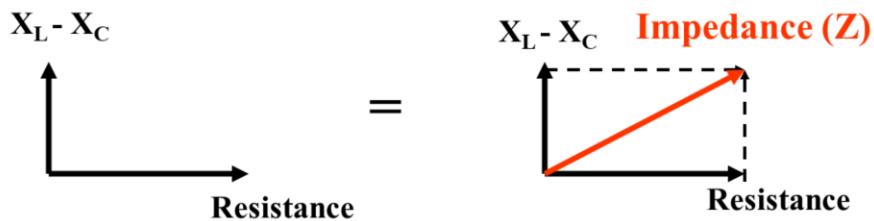
Impedance, denoted Z , is an expression of the opposition that an electronic component, circuit, or system offers to alternating and/or direct electric [current](#). Impedance is a vector (two-dimensional) quantity consisting of two independent scalar (one-dimensional) phenomena: [resistance](#) and [reactance](#).

Resistance, denoted R , is a measure of the extent to which a substance opposes the movement of electrons among its atoms. The more easily the atoms give up and/or accept electrons, the lower the resistance, which is expressed in positive [real number](#) ohms. Resistance is observed with alternating current ([AC](#)) and also with direct current (DC). Examples of materials with low resistance, known as electrical conductors, include copper, silver, and gold. High-resistance substances are called insulators or dielectrics, and include materials such as polyethylene, mica, and glass.

Reactance, denoted X , is an expression of the extent to which an electronic component, circuit, or system stores and releases energy as the current and voltage fluctuate with each AC cycle. Reactance is expressed in [imaginary number](#) ohms. It is observed for AC, but not for DC. When AC passes through a component that contains reactance, energy might be stored and released in the form of a magnetic field, in which case the reactance is inductive (denoted $+jX_L$); or energy might be stored and released in the form of an electric field, in which case the reactance is capacitive (denoted $-jX_C$).

Vector Addition

Through the use of **vector addition**, the Impedance can be determined...



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Vector addition is the operation of adding two or more **vectors** together into a **vector sum**. The so-called parallelogram law gives the rule for **vector addition** of two or more **vectors**. For two **vectors** and , the **vector sum** is obtained by placing them head to tail and drawing the **vector** from the free tail to the free head.

LCR Circuit Impedance Formula

- Rather than plot vectors every time we need to determine impedance however, we can use a **formula**:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Note that because the difference between X_L and X_C is squared, **it doesn't matter** what term is subtracted from what – you can use $X_C - X_L$ if that is more convenient.

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This is just the Pythagorean Theorem rearranged ($Z^2 = R^2 + (X_L - X_C)^2$)

LCR Circuit Impedance Example

- Resistance = 120 Ohms
- $X_L = 40$ Ohms
- $X_C = 130$ Ohms

- $Z = \text{Sqr Root} [R^2 + (X_C - X_L)^2]$

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- $X_L = 40$ Ohms
- $X_C = 130$ Ohms

- $Z = \text{Sqr Root} [R^2 + (X_C - X_L)^2]$
= Sqr Root [$(120)^2 + (130 - 40)^2$]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- $X_L = 40$ Ohms
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- $Z = \text{Sqr Root} [R^2 + (X_C - X_L)^2]$
= Sqr Root [$(120)^2 + (130 - 40)^2$]
= Sqr Root [$14400 + 8100$]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- $X_L = 40$ Ohms
- $X_C = 130$ Ohms

- $Z = \text{Sqr Root} [R^2 + (X_C - X_L)^2]$
= Sqr Root [$(120)^2 + (130 - 40)^2$]
= Sqr Root [$14400 + 8100$]
= Sqr Root [22500]

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LCR Circuit Impedance Example

- Resistance = 120 Ohms
- $X_L = 40$ Ohms
- $X_C = 130$ Ohms

- $Z = \text{Sqr Root} [R^2 + (X_C - X_L)^2]$
= Sqr Root [$(120)^2 + (130 - 40)^2$]
= Sqr Root [$14400 + 8100$]
= Sqr Root [22500]
= 150 Ohms

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LCR Circuit Impedance Example

$X_L = 40 \text{ Ohms}$

Resistance = 120 Ohms

$X_C = 130 \text{ Ohms}$

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LCR Circuit Impedance Example

$X_L = 40 \text{ Ohms}$

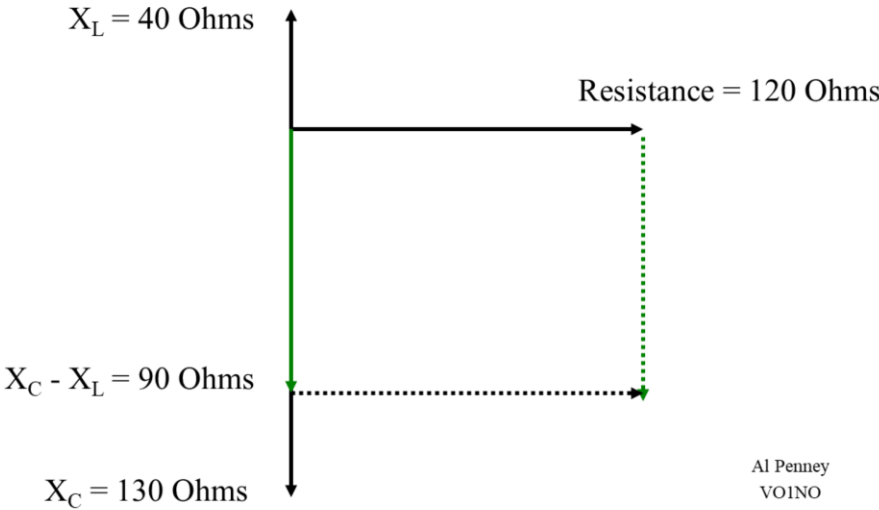
Resistance = 120 Ohms

$X_C - X_L = 90 \text{ Ohms}$

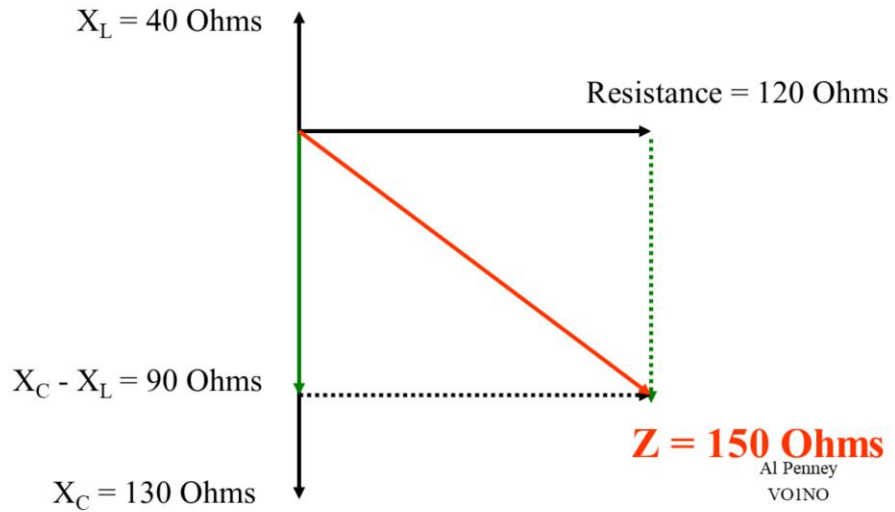
$X_C = 130 \text{ Ohms}$

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LCR Circuit Impedance Example



LCR Circuit Impedance Example



Resonance

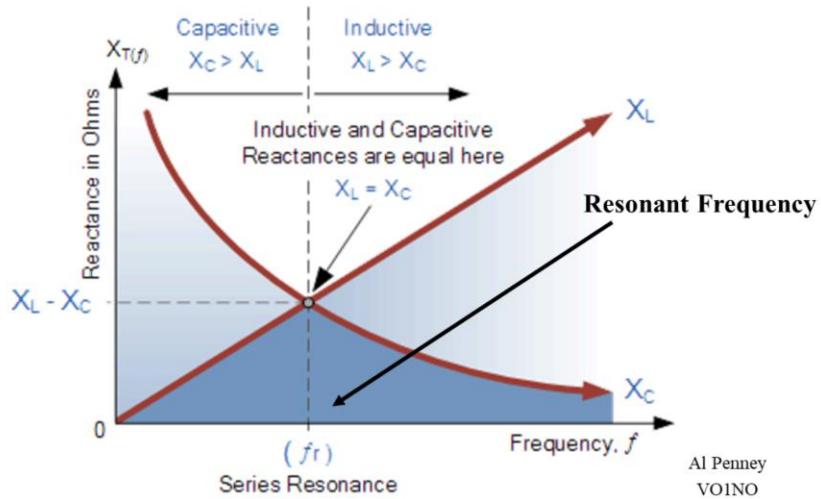
- In electronic circuits, a special condition exists when **Inductive** and **Capacitive Reactance** are **equal** to each other ($X_L = X_C$).
- This condition is known as **Resonance**, and occurs at **only one frequency**, known as the **Resonant Frequency (F_R)**.

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Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is where the two reactance curves cross each other.

Resonance of a circuit involving **capacitors** and **inductors** occurs because the collapsing magnetic field of the inductor generates an electric current in its windings that charges the capacitor, and then the discharging capacitor provides an electric current that builds the magnetic field in the inductor. This process is repeated continually. An analogy is a mechanical **pendulum** and both are a form of **simple harmonic oscillator**.

Resonant Frequency



Resonant Frequency

- At Resonance, $X_C = X_L$ so

$$X_C = \frac{1}{2\pi f C} = X_L = 2\pi f L$$

- With a little mathematical wizardry, we can rearrange that equation to determine the **Resonant Frequency F_R** as follows...

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Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

- Where:

F_R = Resonant Frequency in Hertz

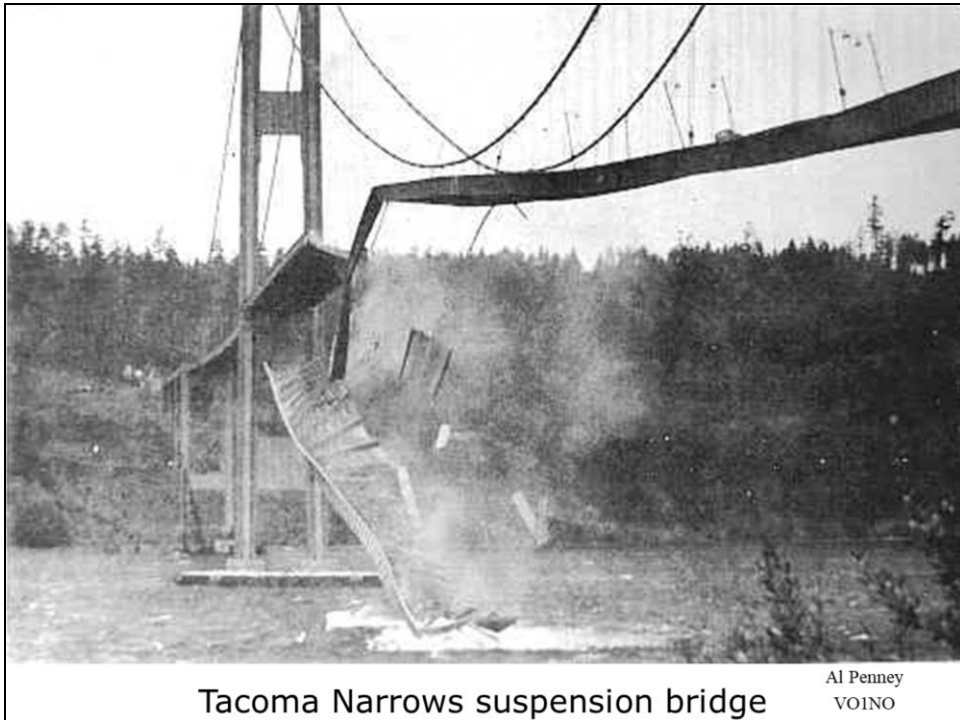
L = Inductance in henrys

C = Capacitance in Farads

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**Resonance is not always a good
thing however...**

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The 1940 Tacoma Narrows Bridge, the first [Tacoma Narrows Bridge](#), was a [suspension bridge](#) in the U.S. state of [Washington](#) that spanned the [Tacoma Narrows strait of Puget Sound](#) between [Tacoma](#) and the [Kitsap Peninsula](#). It opened to traffic on July 1, 1940, and dramatically [collapsed](#) into Puget Sound on November 7 the same year. The bridge's collapse has been described as "spectacular" and in subsequent decades "has attracted the attention of engineers, physicists, and mathematicians". Throughout its short existence, it was the world's third-longest suspension bridge by main span, behind the [Golden Gate Bridge](#) and the [George Washington Bridge](#).

Construction began in September 1938. From the time the [deck](#) was built, it began to move vertically in windy conditions, so construction workers nicknamed the bridge **Galloping Gertie**. The motion continued after the bridge opened to the public, despite several damping measures. The bridge's main span finally collapsed in 40-mile-per-hour (64 km/h) winds on the morning of November 7, 1940, as the deck oscillated in an alternating twisting motion that gradually increased in amplitude until the deck tore apart.

The portions of the bridge still standing after the collapse, including the towers and cables, were dismantled and sold as scrap metal. Efforts to replace the bridge were delayed by the United States' entry into [World](#)

War II, but in 1950, a new Tacoma Narrows Bridge opened in the same location, using the original bridge's tower pedestals and cable anchorages. The portion of the bridge that fell into the water now serves as an artificial reef.

The bridge's collapse had a lasting effect on science and engineering. In many physics textbooks, the event is presented as an example of elementary forced resonance, but it was more complicated in reality; the bridge collapsed because moderate winds produced aeroelastic flutter that was self-exciting and unbounded: for any constant sustained wind speed above about 35 mph (56 km/h), the amplitude of the (torsional) flutter oscillation would continuously increase, with a negative damping factor (i.e. a reinforcing effect, opposite to damping). The collapse boosted research into bridge aerodynamics-aeroelastics, which has influenced the designs of all later long-span bridges.

Tuned or Resonant Circuits

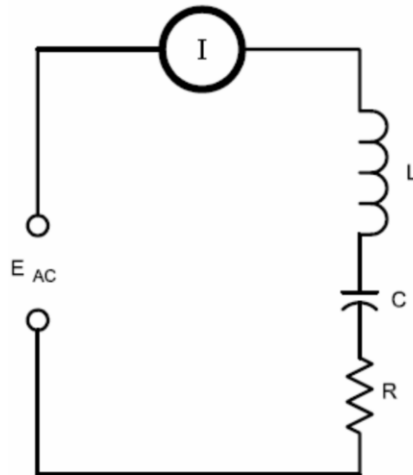
- Circuits containing **Capacitors and Inductors** are often referred to as **Tuned Circuits**.
- They have **many uses** in electronics – every time you tune a radio, you are varying the **resonant frequency** of a **tuned circuit** to **select the signal** you want.

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A **tuned circuit**, is an **electric circuit** consisting of an **inductor**, represented by the letter L, and a **capacitor**, represented by the letter C, connected together. The circuit can act as an electrical **resonator**, an electrical analogue of a **tuning fork**, storing energy oscillating at the circuit's **resonant frequency**.

LC circuits are used either for generating signals at a particular frequency, or picking out a signal at a particular frequency from a more complex signal; this function is called a **bandpass filter**. They are key components in many electronic devices, particularly radio equipment, used in circuits such as **oscillators**, **filters**, **tuners** and **frequency mixers**.

Series LCR Circuit



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Series RLC circuits consist of a resistance, a capacitance and an inductance connected in series across an alternating supply.

Series LCR Circuit

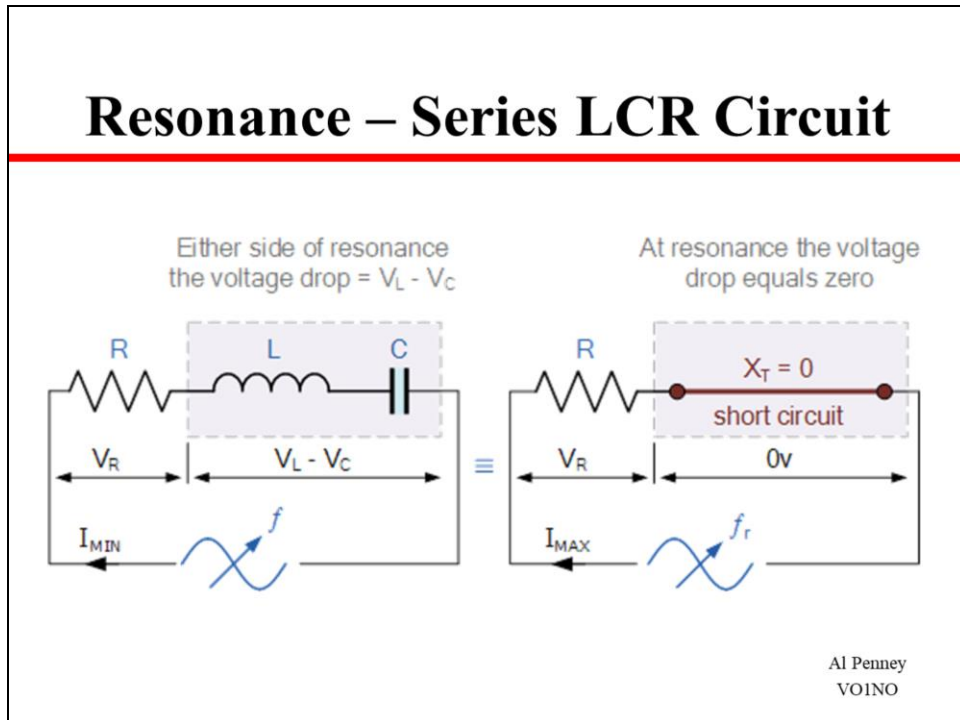
- When a **Series LCR** circuit is in **Resonance**, **current** in that circuit is at its **greatest** (the **Impedance** is at its **lowest**).
- Outside the resonant frequency, the impedance is high, and current therefore low.
- Purpose of a Series LCR circuit is to pass current at the resonant frequency and reject other frequencies.

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In a series RLC circuit there becomes a frequency point where the inductive reactance of the inductor becomes equal in value to the capacitive reactance of the capacitor. In other words, $X_L = X_C$. The point at which this occurs is called the **Resonant Frequency** point, (f_r) of the circuit, and as we are analysing a series RLC circuit this resonance frequency produces a **Series Resonance**.

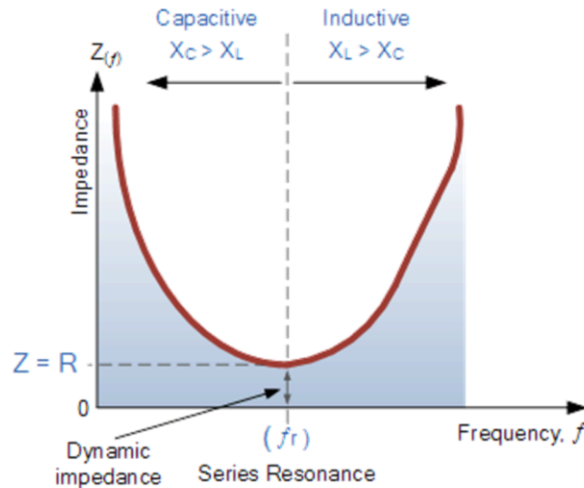
Series Resonance circuits are one of the most important circuits used electrical and electronic circuits. They can be found in various forms such as in AC mains filters, noise filters and also in radio and television tuning circuits producing a very selective tuning circuit for the receiving of the different frequency channels.

Resonance – Series LCR Circuit



We can see then that at resonance, the two reactances cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R . In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely “real”, that is no imaginary impedance’s exist. This is because at resonance they are cancelled out. So the total impedance of the series circuit becomes just the value of the resistance and therefore: $Z = R$. Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit.

Series LCR Circuit Impedance



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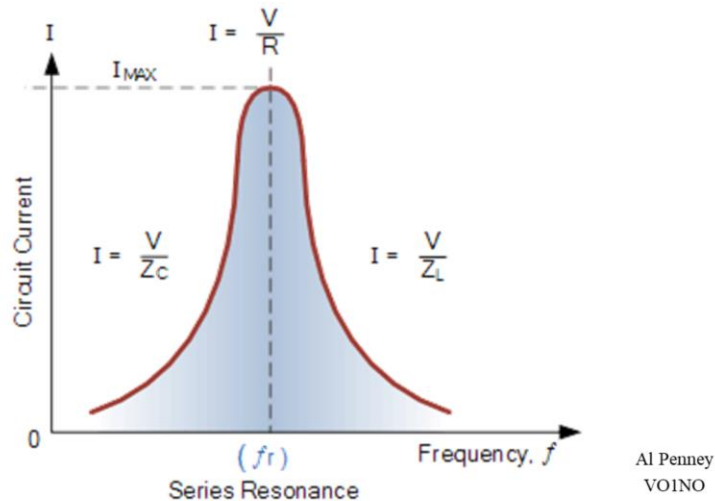
Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L .

You may also note that if the circuit's impedance is at its minimum at resonance then consequently, the circuit's **admittance** must be at its maximum and one of the characteristics of a series resonance circuit is that admittance is very high. But this can be a bad thing because a very low value of resistance at resonance means that the resulting current flowing through the circuit may be dangerously high.

We recall from the previous tutorial about series RLC circuits that the voltage across a series combination is the phasor sum of V_R , V_L and V_C . Then if at resonance the two reactances are equal and cancelling, the two voltages representing V_L and V_C must also be opposite and equal in value thereby cancelling each other out because with pure components the phasor voltages are drawn at $+90^\circ$ and -90° respectively.

Then in a **series resonance** circuit as $V_L = -V_C$ the resulting reactive voltages are zero and all the supply voltage is dropped across the resistor. Therefore, $V_R = V_{\text{supply}}$ and it is for this reason that series resonance circuits are known as voltage resonance circuits, (as opposed to parallel resonance circuits which are current resonance circuits).

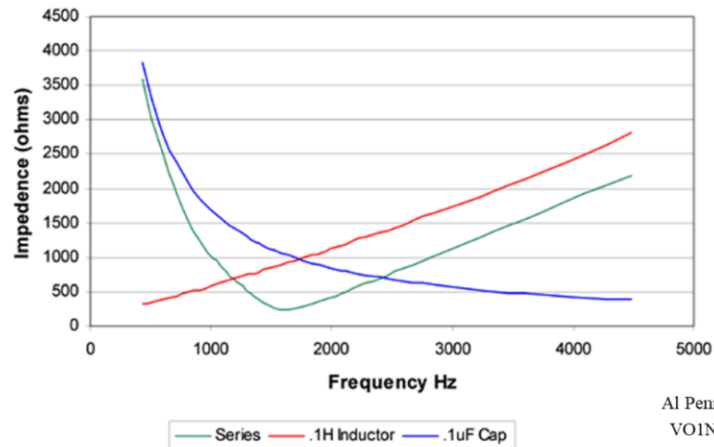
Series Circuit Current



The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite. The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition they cancel each other out.

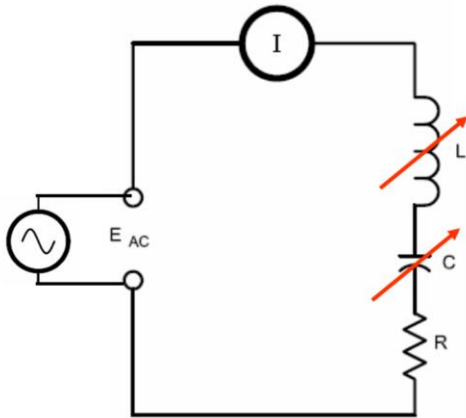
Series LCR Circuit Impedance

L/C in Series .1H/.1uF

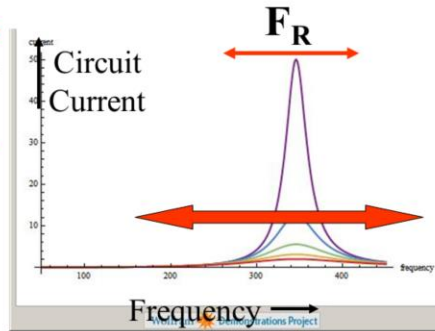


Note that when the capacitive reactance dominates the circuit the impedance curve has a hyperbolic shape to itself, but when the inductive reactance dominates the circuit the curve is non-symmetrical due to the linear response of X_L .

Varying Capacitance or Inductance

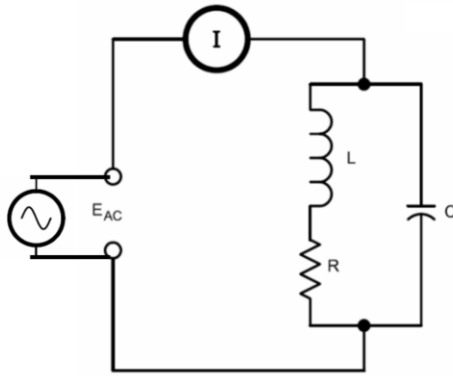


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Parallel LCR Circuits



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RLC Parallel circuit is a circuit in which all the components are connected in parallel across the alternating current source. **There is usually no actual resistor in such a circuit, but the inductor itself has some resistance.**

Parallel LCR Circuits

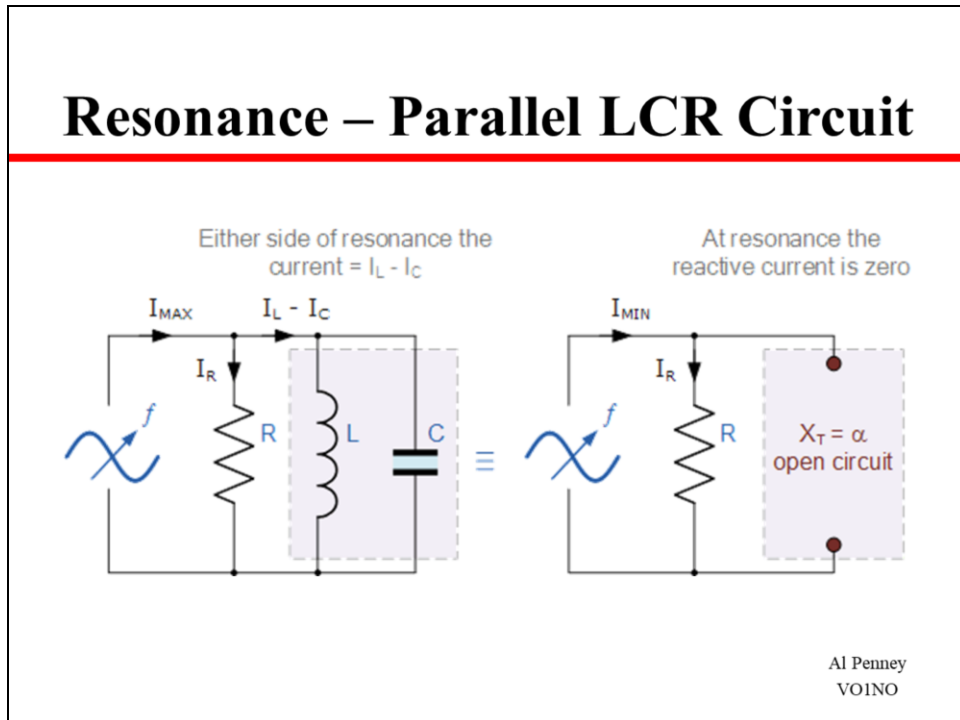
- In a **Parallel LCR Circuit**, the inductor and capacitor are connected in parallel.
- The **current is lowest at Resonance** (the **impedance** is at its **highest**).
- Parallel LCR circuits are used to **reject a specific frequency** while allowing all others to pass.
- Sometimes called a Tank Circuit.

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A parallel circuit containing a resistance, R , an inductance, L and a capacitance, C will produce a **parallel resonance** (also called anti-resonance) circuit when the resultant current through the parallel combination is in phase with the supply voltage. At resonance there will be a large circulating current between the inductor and the capacitor due to the energy of the oscillations, then parallel circuits produce current resonance.

A *parallel resonant circuit* stores the circuit energy in the magnetic field of the inductor and the electric field of the capacitor. This energy is constantly being transferred back and forth between the inductor and the capacitor which results in zero current and energy being drawn from the supply.

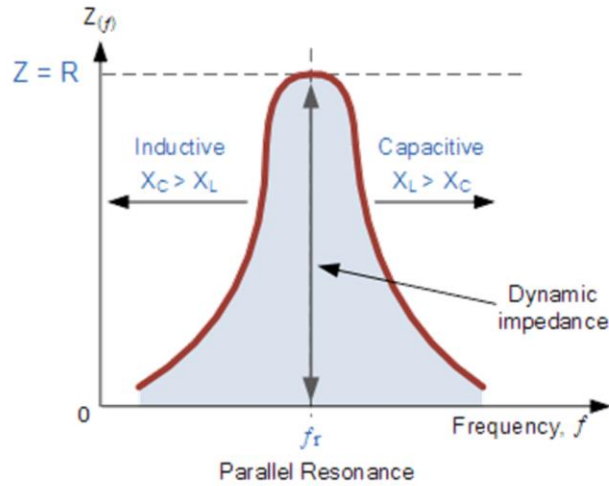
Resonance – Parallel LCR Circuit



Also at resonance the parallel LC tank circuit acts like an open circuit with the circuit current being determined by the resistor, R only. So the total impedance of a parallel resonance circuit at resonance becomes just the value of the resistance in the circuit and $Z = R$ as shown.

Thus at resonance, the impedance of the parallel circuit is at its maximum value and equal to the resistance of the circuit creating a circuit condition of high resistance and low current. Also at resonance, as the impedance of the circuit is now that of resistance only, the total circuit current, I will be “in-phase” with the supply voltage, V_S .

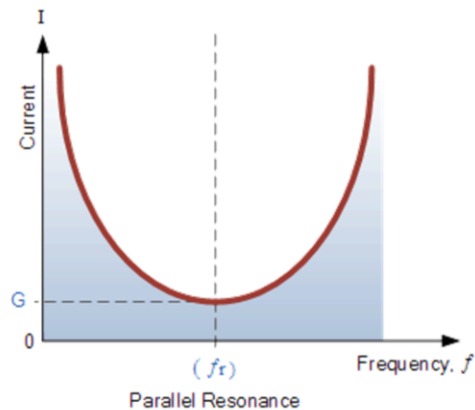
Parallel LCR Circuit Impedance



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In a parallel LCR circuit, impedance is **HIGHEST** at resonance.

Parallel LCR Circuit Current



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The frequency response curve of a parallel resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at its maximum value, reaches its minimum value at the resonance frequency when $I_{\text{MIN}} = I_{\text{R}}$ and then increases again to maximum as f becomes infinite.

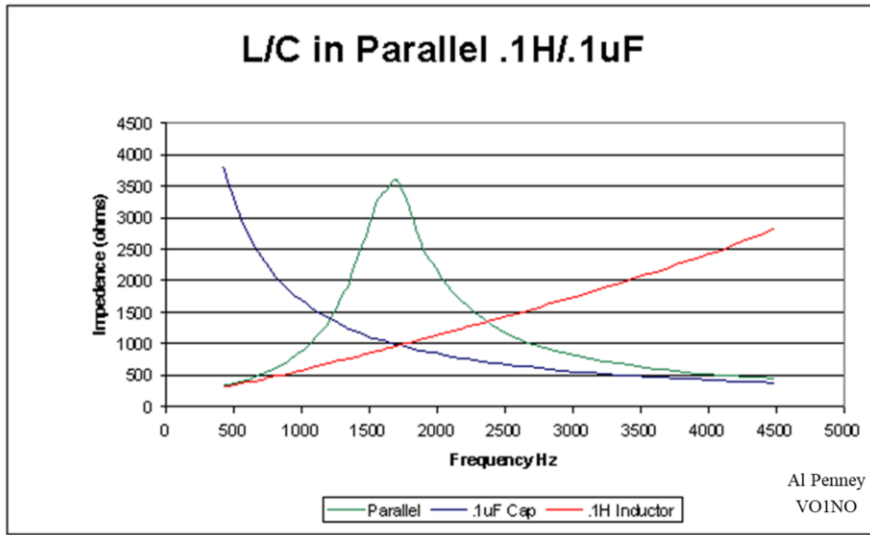
The result of this is that the magnitude of the current flowing through the inductor, L and the capacitor, C tank circuit can become many times larger than the supply current, even at resonance but as they are equal and at opposition (180° out-of-phase) they effectively cancel each other out.

As a parallel resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Rejecter Circuit** because at resonance, the impedance of the circuit is at its maximum thereby suppressing or rejecting the current whose frequency is equal to its resonant frequency. The effect of resonance in a parallel circuit is also called “current resonance”.

The calculations and graphs used above for defining a parallel resonance circuit are similar to those we used for a series circuit. However, the characteristics and graphs drawn for a parallel circuit are exactly opposite to that of series circuits with the parallel circuits

maximum and minimum impedance, current and magnification being reversed. Which is why a parallel resonance circuit is also called an **Anti-resonance** circuit.

Parallel LCR Circuit Impedance



Circuit Quality

- The **Quality** or “**Q**” of a circuit is a **measure of the “sharpness” of the circuit’s selection of frequencies.**
- High Q circuits are necessary in today’s dense RF environment.
- Typical circuit Qs are 50 to 250.

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The Q, or quality, factor of a resonant circuit is a measure of the “goodness” or quality of a resonant circuit. A higher value for this figure of merit corresponds to a more narrow bandwidth, which is desirable in many applications. More formally, Q is the ratio of power stored to power dissipated in the circuit reactance and **resistance**, respectively:

Circuit Quality

- In a **resonant circuit**, energy is stored **alternately** in the **electric field of the capacitor**, and then the **magnetic field of the inductor**.
- This causes a **current** to flow **between them**.
- Anything that **removes energy** from this circuit **broadens the range of frequencies** affected by the circuit, but **increases the impedance** at the **resonant frequency**.

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The resonant current peak may be changed by varying the series resistor, which changes the Q. This also affects the broadness of the curve. A low resistance, high Q circuit has a narrow bandwidth, as compared to a high resistance, low Q circuit.

Series LCR Circuit Quality

- The “**Q**”, or **Quality** of a **Series LCR** circuit is defined as the **ratio** of either X_C or X_L to the **resistance** in the circuit.
- At resonance $X_C = X_L$
- “**Q**” = $X_C / R = X_L / R$

- Note that most of a LCR circuit’s resistance is usually in the inductor windings.

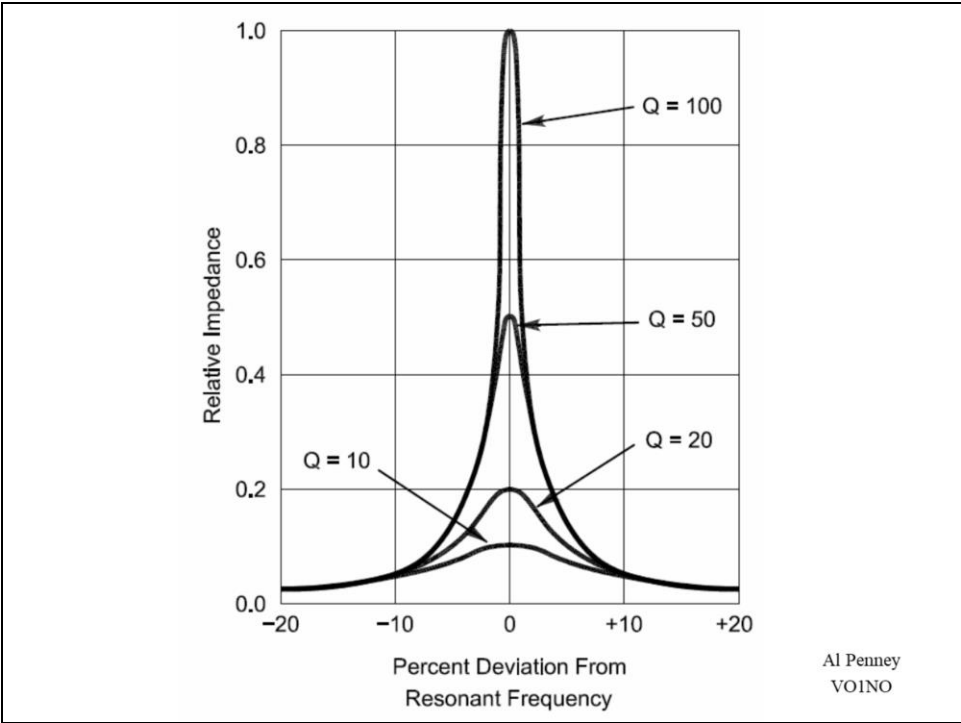
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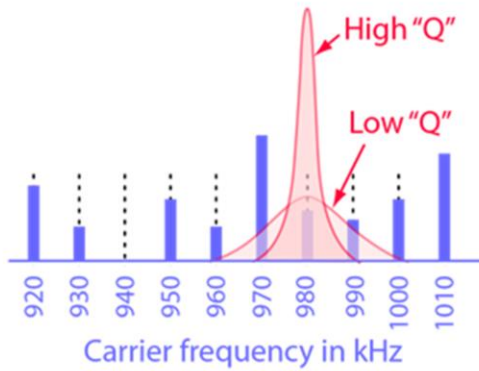
Note that changing the resistance will change the Q of the circuit, but will not change the actual resonant frequency.

Parallel LCR Circuit Quality

- The “**Q**”, or **Quality** of a **Parallel LCR** circuit is defined as the **ratio** of the **resistance** to either **X_C** or **X_L** in the circuit.
- “**Q**” = **$R / X_C = R / X_L$**

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10 kHz bandwidth from
540-1600 kHz for
106 possible bands

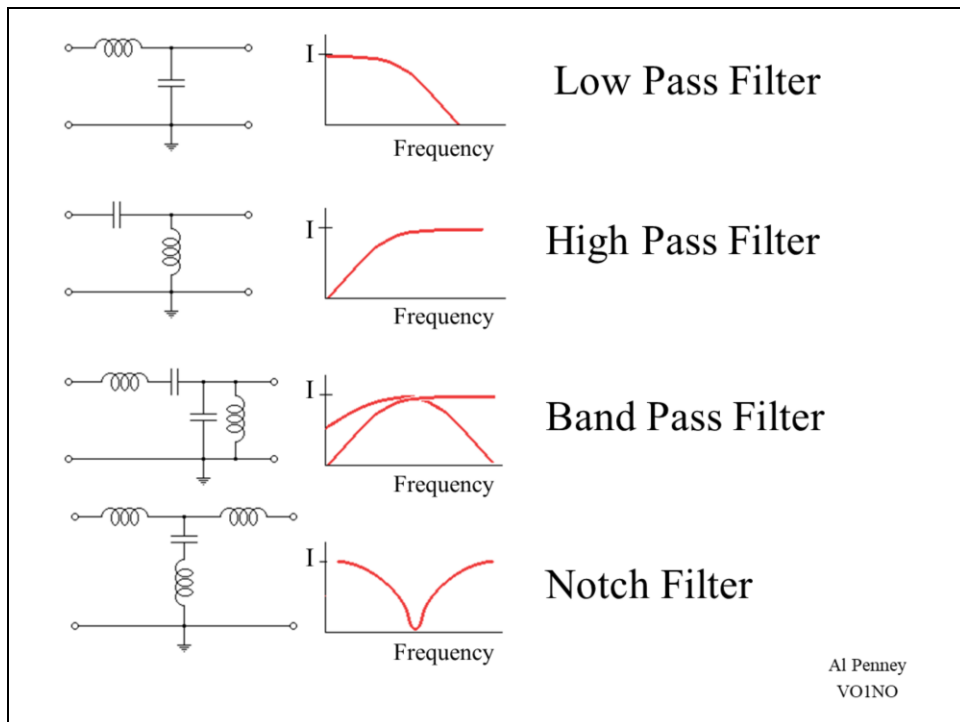
AM Radio

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Filters

- By the proper selection of capacitors and inductors, it is possible to design **Filters** that can **pass desired frequencies**, and **reject unwanted frequencies**.

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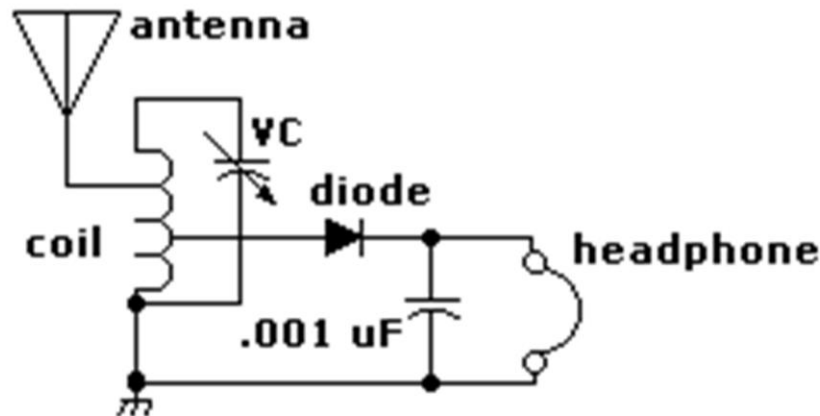
Passive filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or “attenuating” the rest. The most commonly used filter designs are the:

- The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, f_c point to pass while blocking those any higher.
- The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, f_c point and higher to infinity to pass through while blocking those any lower.
- The Band Pass Filter – the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

A Notch filter allows signals above and below a certain frequency to pass through, but attenuates or notches the center frequency.

Crystal Radio Tuned Circuit



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The Q of the tuning circuit in a crystal radio is low, meaning that tuning is very broad. The antenna and ground become part of the tuning circuit, reducing its Q. The diode and headphones draw energy from the tuning section, further reducing its Q.

A crystal radio can be thought of as a radio receiver reduced to its essentials. It consists of at least these components:

- An **antenna** in which **electric currents** are induced by **radio waves**.
- A **resonant circuit** (tuned circuit) which selects the **frequency** of the desired **radio station** from all the radio signals received by the antenna. The tuned circuit consists of a coil of wire (called an **inductor**) and a **capacitor** connected together. The circuit has a **resonant frequency**, and allows radio waves at that frequency to pass through to the detector while largely blocking waves at other frequencies. One or both of the coil or capacitor is adjustable, allowing the circuit to be tuned to different frequencies. In some circuits a capacitor is not used and the antenna serves this function, as an antenna shorter than its resonant length is capacitive.
- A **semiconductor crystal detector** that **demodulates** the radio signal to extract the **audio signal (modulation)**. The crystal detector functions as

a **square law detector**, demodulating the radio frequency **alternating current** to its audio frequency modulation. The detector's audio frequency output is converted to sound by the earphone. Early sets used a "**cat whisker detector**" consisting of a small piece of crystalline mineral such as **galena** with a fine wire touching its surface. The **crystal detector** was the component that gave crystal radios their name. Modern sets use modern **semiconductor diodes**, although some hobbyists still experiment with crystal or other detectors.

- An **earphone** to convert the audio signal to sound waves so they can be heard. The low power produced by a crystal receiver is insufficient to power a **loudspeaker**, hence earphones are used.

Pictorial diagram from 1922 showing the circuit of a crystal radio. This common circuit did not use a tuning **capacitor**, but used the capacitance of the antenna to form the **tuned circuit** with the coil. The detector was a **cat whisker detector**, consisting of a piece of galena with a thin wire in contact with it on a part of the crystal, making a diode contact

As a crystal radio has no power supply, the sound power produced by the earphone comes solely from the **transmitter** of the radio station being received, via the radio waves captured by the antenna. The power available to a receiving antenna decreases with the square of its distance from the **radio transmitter**. Even for a powerful commercial **broadcasting station**, if it is more than a few miles from the receiver the power received by the antenna is very small, typically measured in **microwatts** or **nanowatts**. In modern crystal sets, signals as weak as 50 **picowatts** at the antenna can be heard. Crystal radios can receive such weak signals without using **amplification** only due to the great sensitivity of human **hearing**, which can detect sounds with an intensity of only 10^{-16} W/cm^2 . Therefore, crystal receivers have to be designed to convert the energy from the radio waves into sound waves as efficiently as possible. Even so, they are usually only able to receive stations within distances of about 25 miles for **AM broadcast** stations, although the **radiotelegraphy** signals used during the **wireless telegraphy** era could be received at hundreds of miles, and crystal receivers were even used for transoceanic communication during that period.

Tuned Circuit:

The **tuned circuit**, consisting of a coil and a **capacitor** connected together, acts as a **resonator**, similar to a tuning fork. Electric charge, induced in the antenna by the radio waves, flows rapidly back and forth between the plates of the capacitor through the coil. The circuit has a high **impedance** at the desired radio signal's frequency, but a low impedance at all other frequencies. Hence,

signals at undesired frequencies pass through the tuned circuit to ground, while the desired frequency is instead passed on to the detector (diode) and stimulates the earpiece and is heard. The frequency of the station received is the **resonant frequency** f of the tuned circuit, determined by the **capacitance** C of the capacitor and the **inductance** L of the coil.

The circuit can be adjusted to different frequencies by varying the inductance (L), the capacitance (C), or both, "tuning" the circuit to the frequencies of different radio stations. In the lowest-cost sets, the inductor was made variable via a spring contact pressing against the windings that could slide along the coil, thereby introducing a larger or smaller number of turns of the coil into the circuit, varying the **inductance**. Alternatively, a **variable capacitor** is used to tune the circuit. Some modern crystal sets use a **ferrite core** tuning coil, in which a ferrite **magnetic core** is moved into and out of the coil, thereby varying the inductance by changing the **magnetic permeability** (this eliminated the less reliable mechanical contact).

The antenna is an integral part of the tuned circuit and its **reactance** contributes to determining the circuit's resonant frequency. Antennas usually act as a **capacitance**, as antennas shorter than a quarter-wavelength have **capacitive reactance**. Many early crystal sets did not have a tuning capacitor, and relied instead on the capacitance inherent in the wire antenna (in addition to significant **parasitic capacitance** in the coil) to form the tuned circuit with the coil.

The earliest crystal receivers did not have a tuned circuit at all, and just consisted of a crystal detector connected between the antenna and ground, with an earphone across it. Since this circuit lacked any frequency-selective elements besides the broad **resonance** of the antenna, it had little ability to reject unwanted stations, so all stations within a wide band of frequencies were heard in the earphone (in practice the most powerful usually drowns out the others). It was used in the earliest days of radio, when only one or two stations were within a crystal set's limited range.

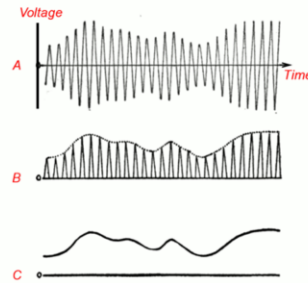
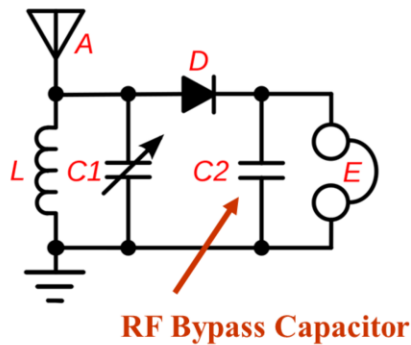
Impedance Matching:

An important principle used in crystal radio design to transfer maximum power to the earphone is **impedance matching**. The maximum power is transferred from one part of a circuit to another when the **impedance** of one circuit is the complex conjugate of that of the other; this implies that the two circuits should have equal resistance. However, in crystal sets, the impedance of the antenna-ground system (around 10-200 **ohms**) is usually lower than the

impedance of the receiver's tuned circuit (thousands of ohms at resonance), and also varies depending on the quality of the ground attachment, length of the antenna, and the frequency to which the receiver is tuned.

Therefore, in improved receiver circuits, in order to match the antenna impedance to the receiver's impedance, the antenna was connected across only a portion of the tuning coil's turns. This made the tuning coil act as an **impedance matching transformer** (in an **autotransformer** connection) in addition to providing the tuning function. The antenna's low resistance was increased (transformed) by a factor equal to the square of the turns ratio (the ratio of the number of turns the antenna was connected to, to the total number of turns of the coil), to match the resistance across the tuned circuit. In the "two-slider" circuit, popular during the wireless era, both the antenna and the detector circuit were attached to the coil with sliding contacts, allowing (interactive) adjustment of both the resonant frequency and the turns ratio. Alternatively a multiposition switch was used to select taps on the coil. These controls were adjusted until the station sounded loudest in the earphone.

RF Bypass Capacitor

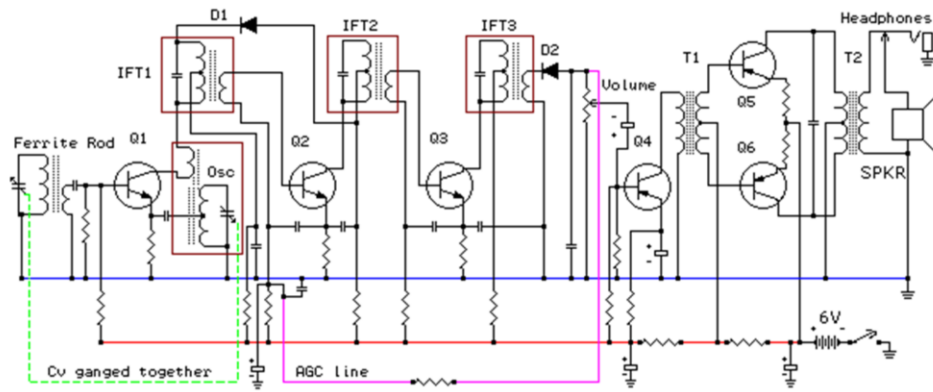


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How the crystal detector works.

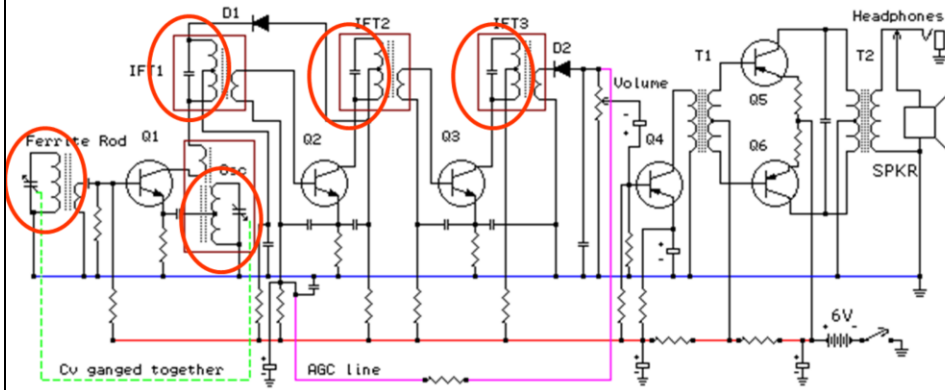
(A) The **amplitude modulated** radio signal from the tuned circuit. The rapid oscillations are the **radio frequency carrier wave**. The **audio signal** (the sound) is contained in the slow variations (**modulation**) of the amplitude (hence the term amplitude modulation, AM) of the waves. This signal cannot be converted to sound by the earphone, because the audio excursions are the same on both sides of the axis, averaging out to zero, which would result in no net motion of the earphone's diaphragm. (B) The crystal conducts current better in one direction than the other, producing a signal whose amplitude does not average to zero but varies with the audio signal. (C) A bypass capacitor is used to remove the radio frequency carrier pulses, leaving the audio signal

6 Transistor Radio



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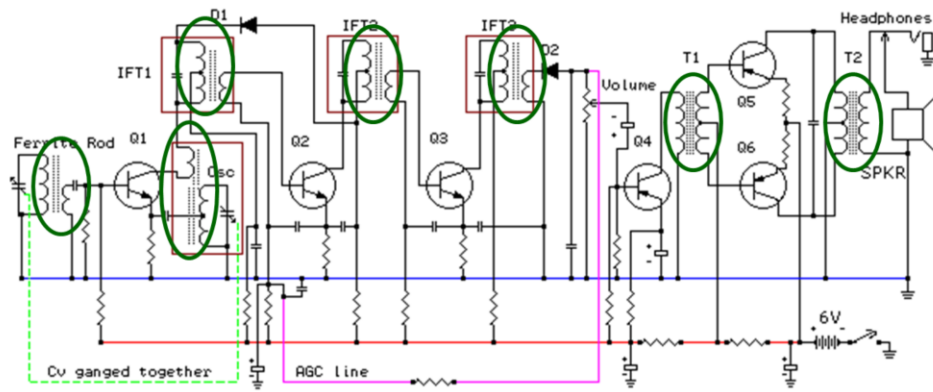
6 Transistor Radio



Tuned Circuits

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6 Transistor Radio



Transformers

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Review Question 1

Resonance is the condition that exists when:

- resistance is equal to the reactance
- inductive reactance and capacitive reactance are equal
- inductive reactance is the only opposition in the circuit
- the circuit contains no resistance

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Review Question 1

Resonance is the condition that exists when:

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- inductive reactance and capacitive reactance are equal
- inductive reactance is the only opposition in the circuit
- the circuit contains no resistance

< inductive reactance and capacitive reactance are equal >

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Review Question 2

Parallel tuned circuits offer:

- high impedance at resonance
- low impedance at resonance
- zero impedance at resonance
- an impedance equal to resistance of the circuit

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Review Question 2

Parallel tuned circuits offer:

- high impedance at resonance
 - low impedance at resonance
 - zero impedance at resonance
 - an impedance equal to resistance of the circuit
- < **high impedance at resonance** >

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Review Question 3

Resonance is an electrical property used to describe:

- an inductor
- a set of parallel inductors
- the results of tuning a varicap (varactor)
- the frequency characteristic of a coil and capacitor circuit

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Review Question 3

Resonance is an electrical property used to describe:

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 - the results of tuning a varicap (varactor)
 - the frequency characteristic of a coil and capacitor circuit
- < **the frequency characteristic of a coil and capacitor circuit**
>

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Review Question 4

In a parallel-resonant circuit at resonance, the circuit has a:

- high impedance
- low impedance
- low mutual inductance
- high mutual inductance

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Review Question 4

In a parallel-resonant circuit at resonance, the circuit has a:

- high impedance
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 - low mutual inductance
 - high mutual inductance
- < **high impedance** >

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Review Question 5

A tuned circuit is formed from two basic components. These are:

- resistors and transistors
- directors and reflectors
- diodes and transistors
- inductors and capacitors

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Review Question 5

A tuned circuit is formed from two basic components. These are:

- resistors and transistors
 - directors and reflectors
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 - inductors and capacitors
- < **inductors and capacitors** >

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Review Question 6

When a parallel coil-capacitor combination is supplied with AC of different frequencies, there will be one frequency where the impedance will be highest. This is the:

- reactive frequency
- resonant frequency
- impedance frequency
- inductive frequency

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Review Question 6

When a parallel coil-capacitor combination is supplied with AC of different frequencies, there will be one frequency where the impedance will be highest. This is the:

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- resonant frequency
- impedance frequency
- inductive frequency
- < **resonant frequency** >

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Review Question 7

In a series resonant circuit at resonance, the circuit has:

- low impedance
- high impedance
- low mutual inductance
- high mutual inductance

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Review Question 7

In a series resonant circuit at resonance, the circuit has:

- low impedance
 - high impedance
 - low mutual inductance
 - high mutual inductance
- < **low impedance** >

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Review Question 8

A coil and an air-spaced capacitor are arranged to form a resonant circuit. The resonant frequency will remain the same if we:

- wind more turns on the coil
- add a resistor to the circuit
- increase the area of plates in the capacitor
- insert Mylar sheets between the plates of the capacitor

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Review Question 8

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 - add a resistor to the circuit
 - increase the area of plates in the capacitor
 - insert Mylar sheets between the plates of the capacitor
- < **add a resistor to the circuit** >

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Review Question 9

Resonant circuits in a receiver are used to:

- select signal frequencies
- filter direct current
- increase power
- adjust voltage levels

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Review Question 9

Resonant circuits in a receiver are used to:

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 - filter direct current
 - increase power
 - adjust voltage levels
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Review Question 10

Resonance is the condition that exists when:

- inductive reactance is the only opposition in the circuit
- the circuit contains no resistance
- resistance is equal to the reactance
- inductive reactance and capacitive reactance are equal and opposite in sign

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Review Question 10

Resonance is the condition that exists when:

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< inductive reactance and capacitive reactance are equal and opposite in sign >

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Review Question 11

When a series LCR circuit is tuned to the frequency of the source, the:

- line current reaches maximum
- line current lags the applied voltage
- line current leads the applied voltage
- impedance is maximum

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Review Question 11

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- line current lags the applied voltage
- line current leads the applied voltage
- impedance is maximum

< line current reaches maximum >

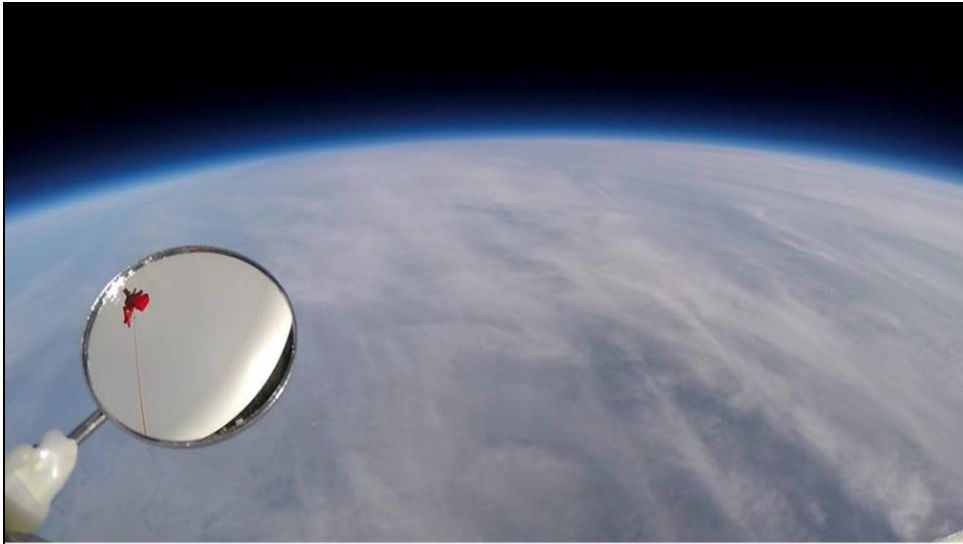
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For Next Class:

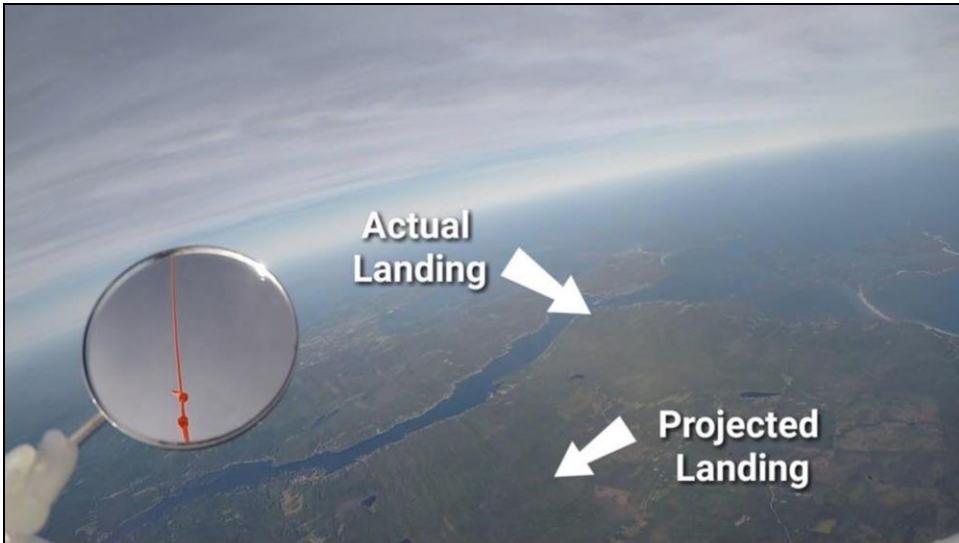
- Review Chapter 4 of Basic Study Guide;
- Read Chapter 5 of Basic Study Guide; and
- Read Radiocommunications Information Circular (RIC-9):
 - <https://www.ic.gc.ca/eic/site/smt-gst.nsf/eng/sf02102.html>

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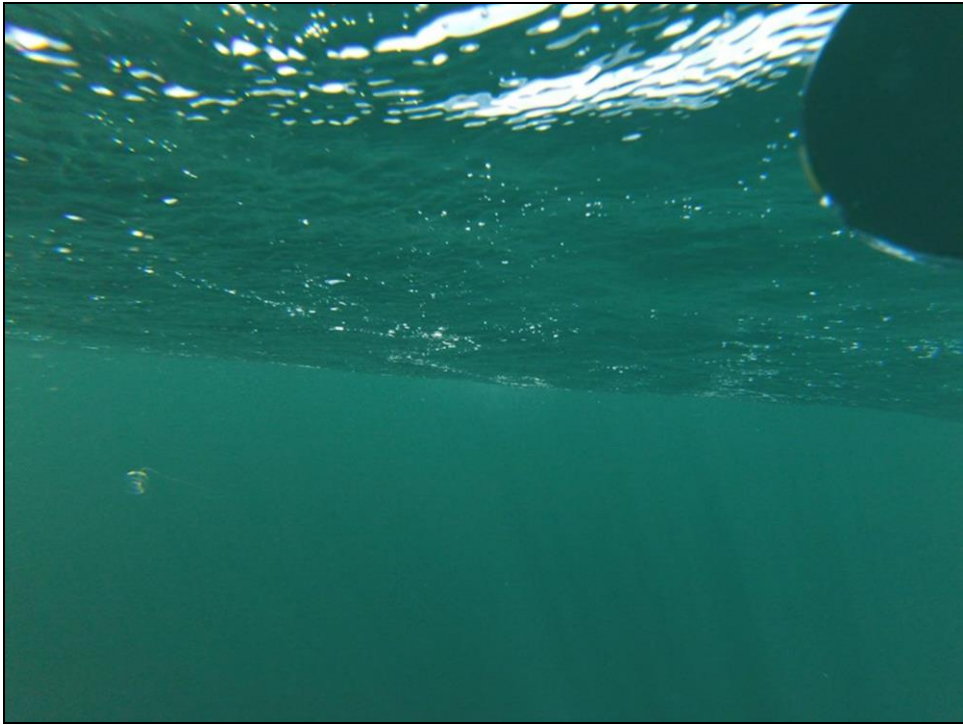


Annapolis Royal Space Agency



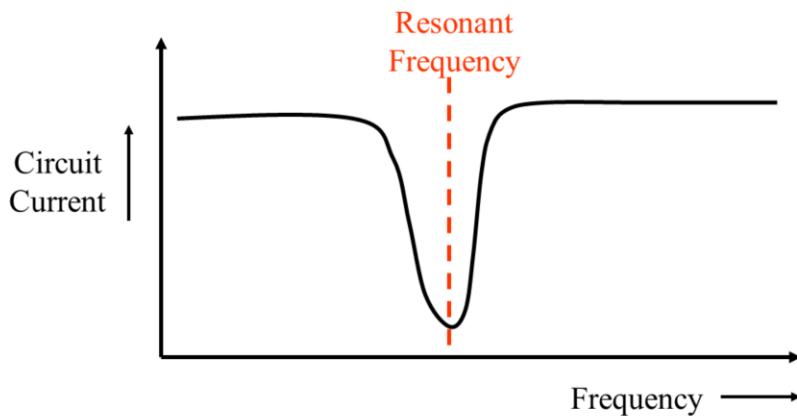
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Parallel LCR Circuit Current



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Resonant Frequency

$$F_R = \frac{1}{2\pi\sqrt{LC}}$$

- Where:

F_R = Resonant Frequency in Hertz

L = Inductance in henrys

C = Capacitance in Farads

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Third time for this slide – **KNOW THIS FORMULA!**

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Another Useful Relationship

- If the circuit Q is known bandwidth can be calculated:
- **$BW = F / Q$**
 - **BW is Bandwidth in Hz**
 - **F is center frequency in Hz**
 - **Q is circuit Q**
- Examples are in Study Guide and Question Bank – **do them!**

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Inductor Q

- The winding resistance of the inductor is usually the greatest resistance in a circuit.
- The Q of an inductor can be calculated if that resistance is known.
- $Q_L = X_L / R_L = 2\pi FL / R_L$

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